How to control hyperbolic systems of first-order equations



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Preamble

- Joint work with
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  - Sonja Steffensen: RWTH-Aachen;
  - Gediyon Y. Weldegiyorgis: formerly of University of Pretoria;
  - Elimboto Yohana: University of Dar es Salaam.

Introduction

Generic one-dimensional hyperbolic system

$$\partial_t Y + \partial_x f(Y) + S(Y, x) = 0$$

OR

$$\partial_t Y + F(Y)\partial_x Y + S(Y, x) = 0$$

- Y is the state of the system;
- $F(Y)$ is the Jacobian of F, a diagonalisable matrix with distinct and real eigenvalues;
- $S(Y, x)$ is the source term;
- $S(Y, x) = 0$ - the system is homogeneous - conservative form;
- On a bounded domain of $x \in [0, l]$ - boundary conditions needed i.e. some information enters the system and some information leaves the systems;

Examples of hyperbolic systems

- Isothermal/Isentropic Euler Equations of Gas Dynamics

$$\begin{aligned} \partial_t \rho + \partial_x(\rho u) &= 0; \\ \partial_t(\rho u) + \partial_x(\rho u^2 + p) &= 0; \end{aligned}$$

  Equation of State: $p = a^2\rho$ (isothermal), $p = \kappa\rho^\gamma$ (isentropic), where a is speed of sound, $\kappa > 0$ constant, $\gamma > 1$.

- Shallow Water Equations

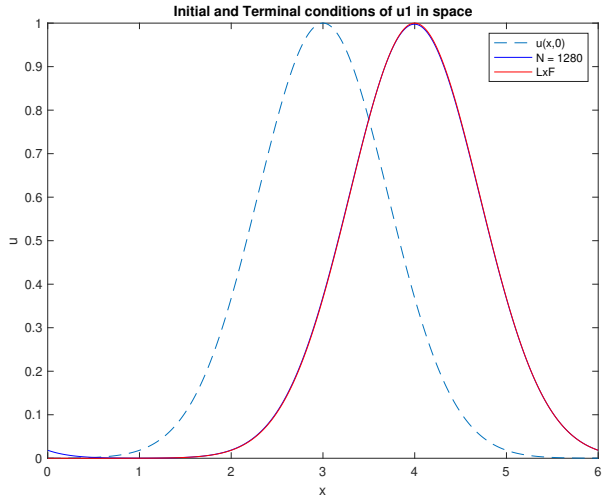
$$\begin{aligned} \partial_t h + \partial_x(hu) &= 0; \\ \partial_t(hu) + \partial_x(hu^2 + \frac{1}{2}gh^2) &= 0. \end{aligned}$$

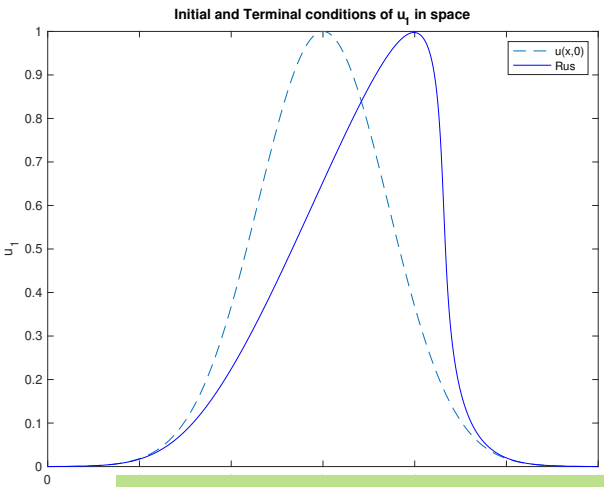
Partial Differential Equations of Hyperbolic Type

$$\partial_t Y + \partial_x f(Y) + S(Y, x) = 0$$

- time dependent - evolutionary models;
- describe transport phenomena;
- finite speed of propagation;
- Examples: traffic flow, gas dynamics, river flow and blood flow (arterial networks).
- *The physics changes, but the mathematics of propagation, conservation and interaction often remains remarkably similar.*

Example: Scalar Linear versus nonlinear models - peculiarities



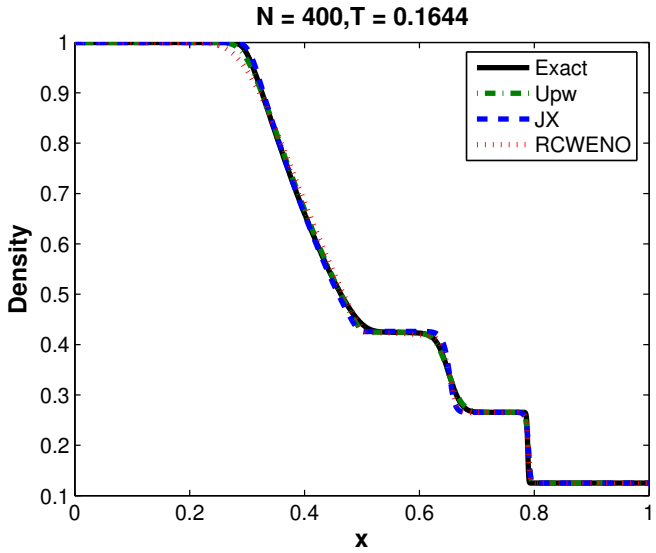


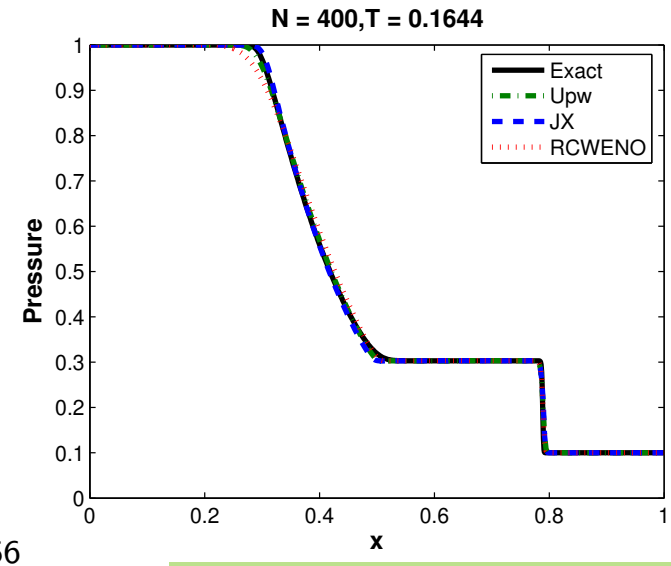
- Advection: $y_t + a y_x = 0$;
- Burgers: $y_t + (\frac{y^2}{2})_x = 0$; with

$$y(x,0) = y_0(x) = e^{-(x-3)^2}$$

- and $x \in [0,6]$, $a = 1$, $T = 1$, CFL $= 0.47693547$;

Riemann Problem: Sod's Problem





- Consider:

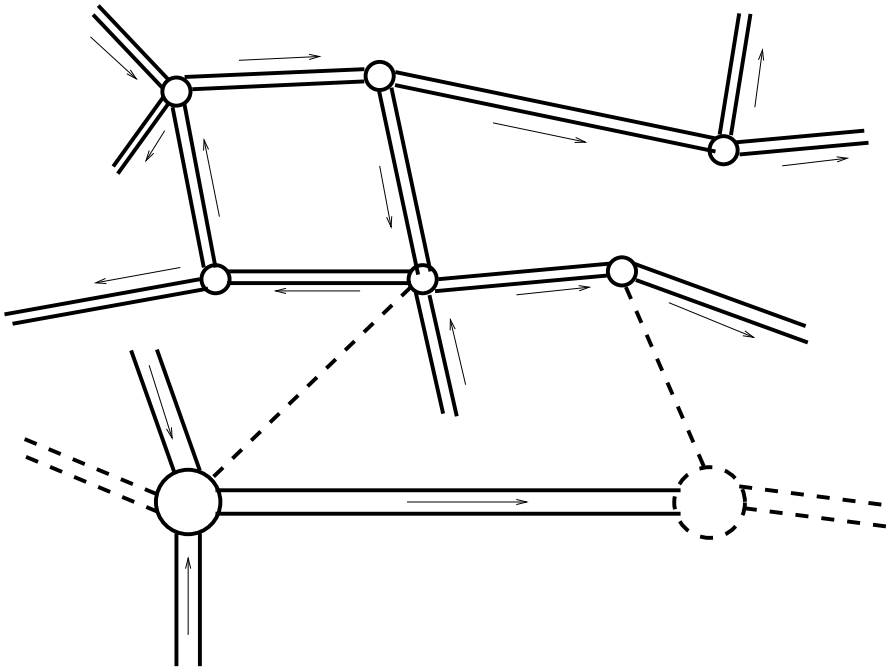
$$y_t + f(y)_x = 0;$$

with

$$y(x,0) = y_0(x) = \begin{cases} y_L & \text{if } x < 0; \\ y_R & \text{if } x \geq 0. \end{cases}$$

- Solutions of the Riemann problem are simple waves:
  - rarefaction waves;
  - shock waves;
  - contact discontinuities.

A flow problem on a Network



- Flow/Transport phenomena along each arc described by a spatial 1-D hyperbolic equation.
- Mathematical description as coupled systems of hyperbolic balance or conservation laws.

Networks: The Riemann problem at a junction

Consider

$$\partial_t y^{(e)} + \partial_x f(y^{(e)}) = S(x,t,y^{(e)}), \quad (x,t) \in \mathbb{R} \times \mathbb{R}^+$$
$$y^{(e)}(x,0) = \bar{y}_0^{(e)}(x), \quad x \in \mathbb{R}.$$

With coupling conditions (interior boundary conditions) on the interface:

$$\Psi(y^{(1)}, \ldots, y^{(N)}) = \Pi(t).$$

Π referred to as coupling constant but dependent on t and $y^{(e)}(x,0) = \bar{y}_0^{(e)}$ are constant states.

Isothermal ($p = a^2\rho$) Pipe-Pipe Connection

Consider:

- incoming wave on Pipe 1, low flux profile on outgoing Pipe 2;
- subsonic conditions at the interface;
- Pipe parameters:

$$f_g = 10^{-3}; \qquad D = 0.1\text{m}$$

- Flow Boundary Conditions:

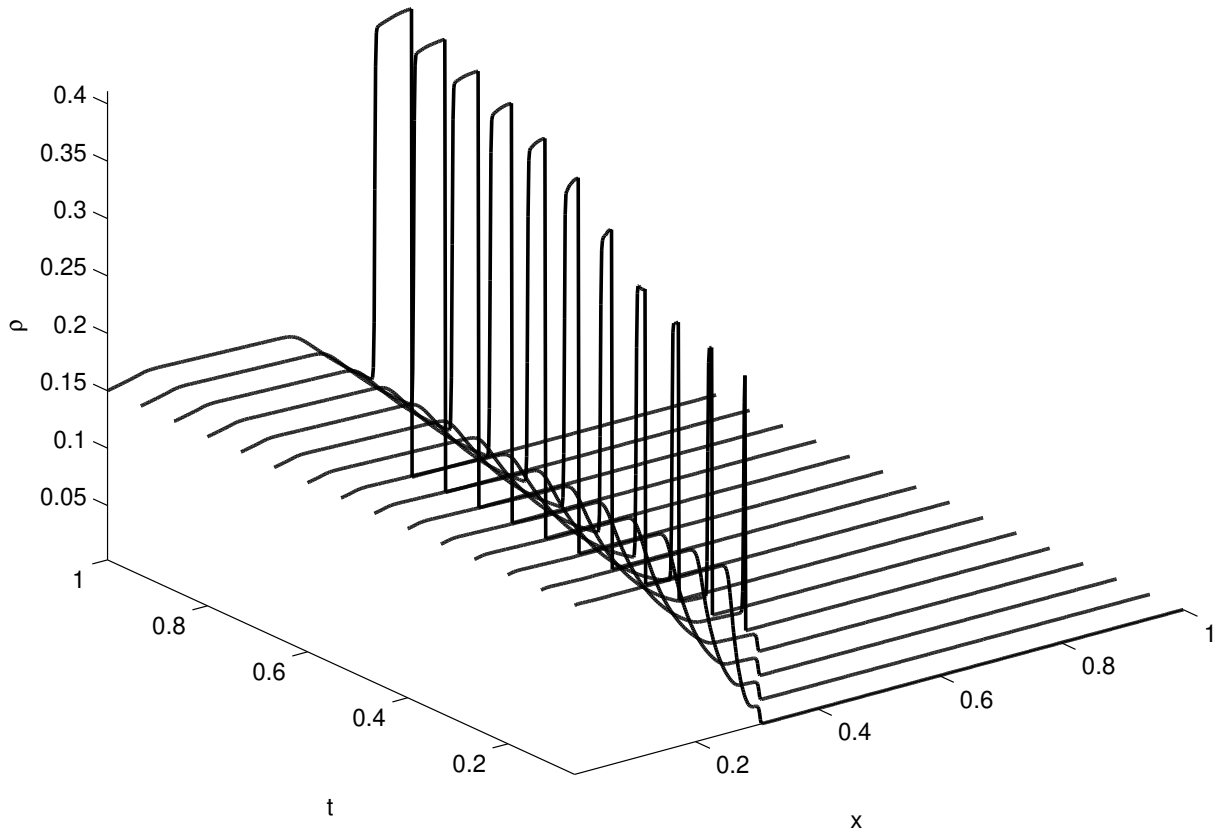
$$q_{\text{in}} = 70\text{Kg m}^{-2}\text{ s}^{-1}, \qquad q_{\text{out}} = 1\text{ Kg m}^{-2}\text{ s}^{-1};$$

- Initial conditions:

$$U^0 = [1/360, 1];$$

- Gas parameter: $a = 360\text{m/s}$.

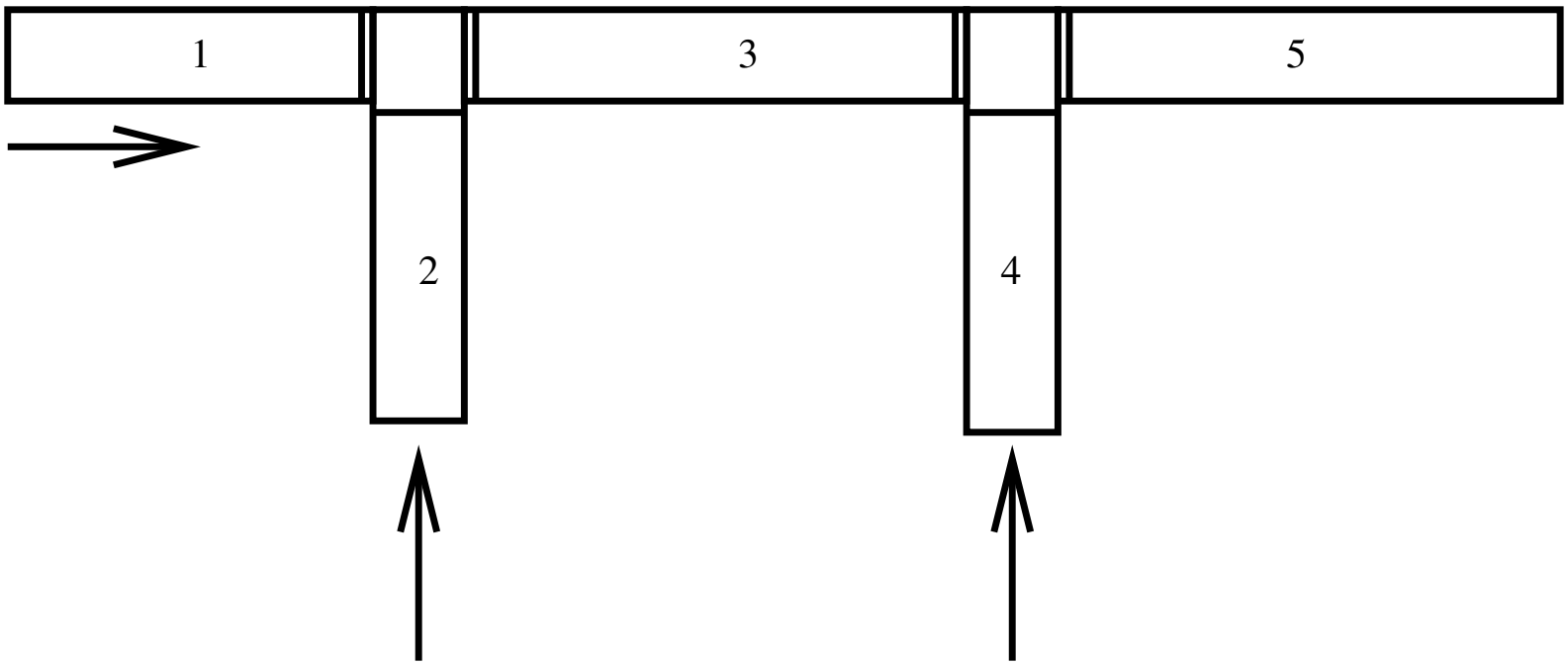
Pipe-Pipe Connection



Note: Backward moving shock develops on Pipe 1, density increase near intersection.

Five Pipes with 'Tee' Junctions

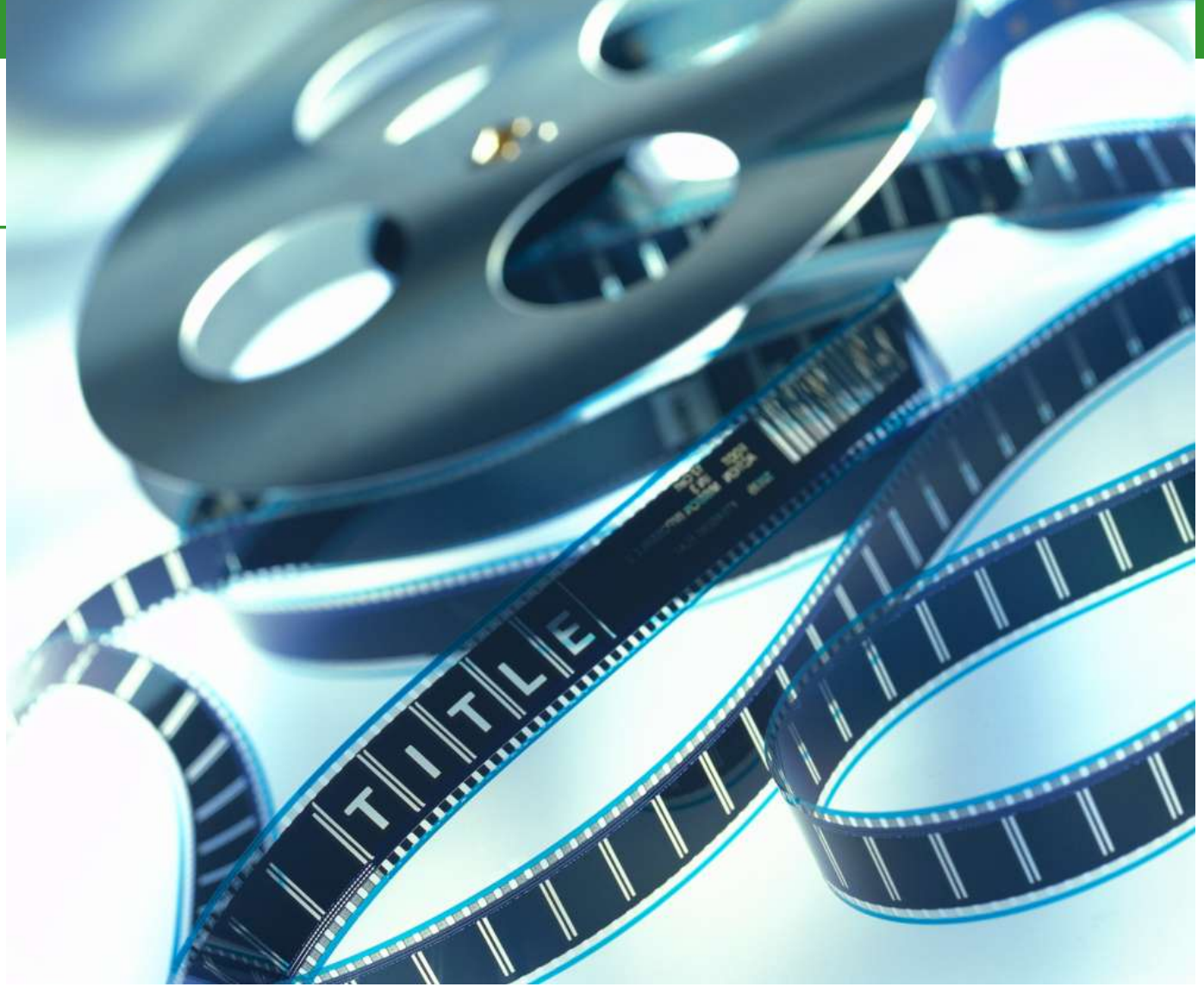
$f_g = 10^{-3}$. $D = 0.1$. $a = 6$, $L = 1$.



Network Setup

initial conditions on pipes 1,3,5: $(4, 2)$, $(4, 4)$, $(4, 6)$.

Pressure





Flow Control

Control - Optimization

Consider the problem

$$\min_w J(w) = \frac{1}{2}\|y(w) - y_d\|_Y^2, \quad \forall w \in \mathcal{U}_{\text{ad}}$$

subject to

$$A(y) = f(w).$$

- Or $J(w) = \frac{1}{2}\|y(w) - y_d\|_Y^2 + \frac{\mu}{2}\|w\|_{\mathcal{U}}^2, \quad \forall w \in \mathcal{U}_{\text{ad}}$ where $\mu \geq 0$.
- Lagrangian using Lagrangian multipliers (adjoint variables)

$$\mathcal{L}(y, w, \lambda) = J(w) - \langle A(y) - f(w), \lambda \rangle.$$

- Regularisation terms with $\mu \geq 0$ leads to improved smoothness properties of optimal control.

Alternative to Linearization - Relaxation



Consider an optimal control problem

$$\min_{y_0} \mathcal{J}(\mathbf{y}(\cdot, T), \mathbf{y}_0, \mathbf{y}_d) \quad \text{subject to} \quad \begin{cases} \partial_t \mathbf{y} + \partial_x \mathbf{f}(\mathbf{y}) = 0, \\ \mathbf{y}(x, 0) = \mathbf{y}_0. \end{cases}$$

- Tracking type problem with $\mathbf{y}_0$ as control variable.
- Use adjoint (λ) identity to evaluate gradients (Steepest Descent Approach) of the cost functional with respect to control variable.
- Apply a semi-linear form - relaxation [B, Herty, etc.].
- Other approaches [Bouchut, James, Bressan, Shen, Ulbrich, Homescu and Navon].

Grid Convergence Example

Constraint: $\partial_t y + \partial_x f(y) = 0$ where $f(y) = cy$.

- $y_0(x) = y_d(x - cT)$, periodic boundary conditions; $c \equiv 1$.
- $x \in [0, 2\pi]$; CFL $= \frac{1}{2}$.
- Rate of convergence,

$$\rho = \ln(E^{2h}/E^h)/\ln(r)$$

where

- ρ - the rate; r the ratio of the grid refinement;
- E are the errors with h the stepwidth.

Grid Convergence Example



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N_x	L_1	Rate
100	9.59e-03	
200	2.66e-03	1.80
400	7.12e-04	1.86
800	1.89e-04	1.88
1600	4.96e-05	1.90

Table: Grid convergence for the second–order adjoint relaxing scheme with exact terminal conditions at $t = T$ and relaxation rate $\tau = 1.e - 08$.

Control of Burgers Equation

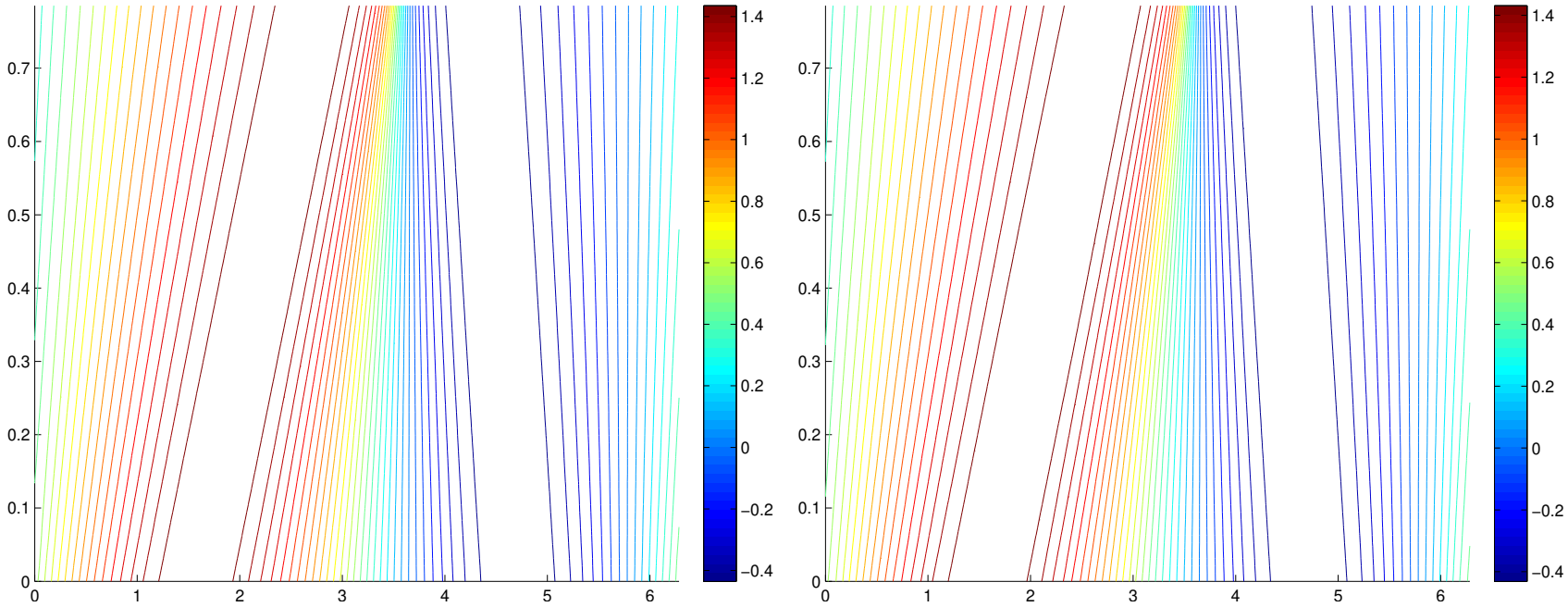
Now the flux function

$$f(y) = \frac{1}{2}y^2$$

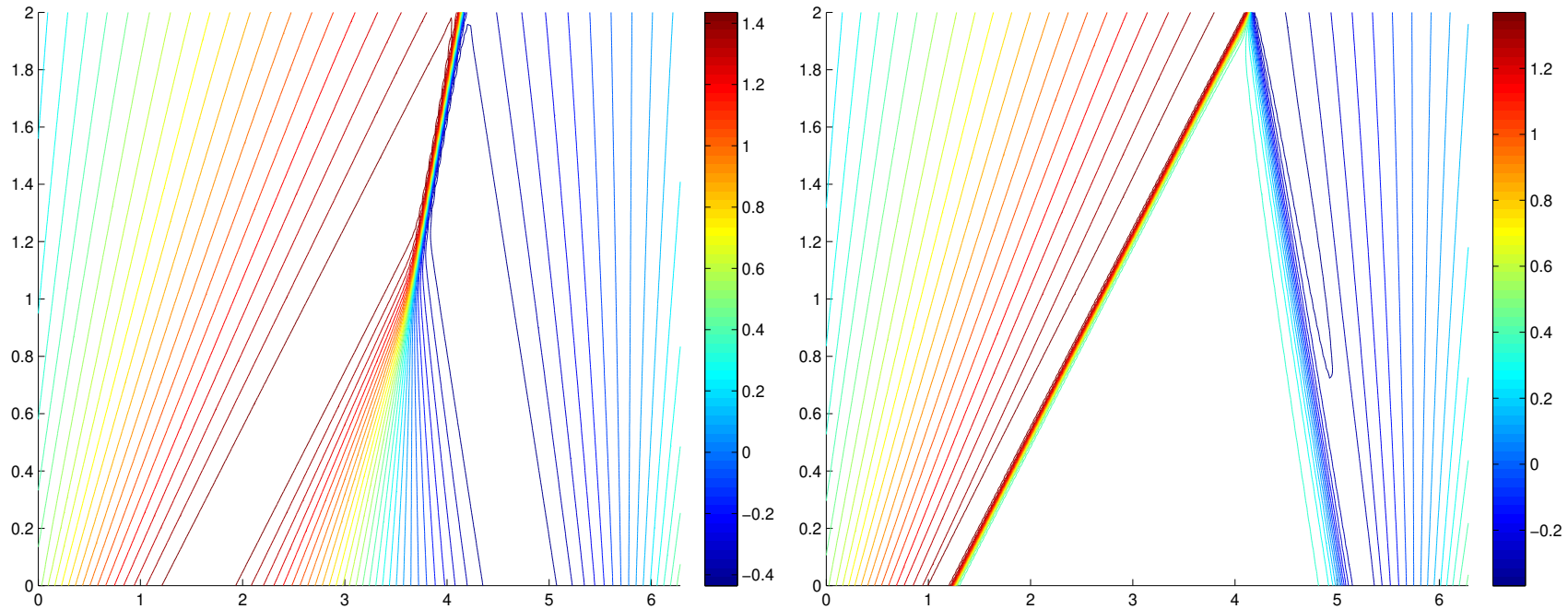
with $y_0(x) = \frac{1}{2} + \sin(x)$ on $x \in [0, 2\pi]$.

- Shock wave of speed $s = 1$ after the critical time $t = \pi/2$.
- Use solutions $y(x, t = 2)$ and $y(x, t = \pi/4)$ as desired states y_d.

Control of Burgers Equation



Control of Burgers Equation - Mind the Gap!



Non-Linear Programming Approach [1]

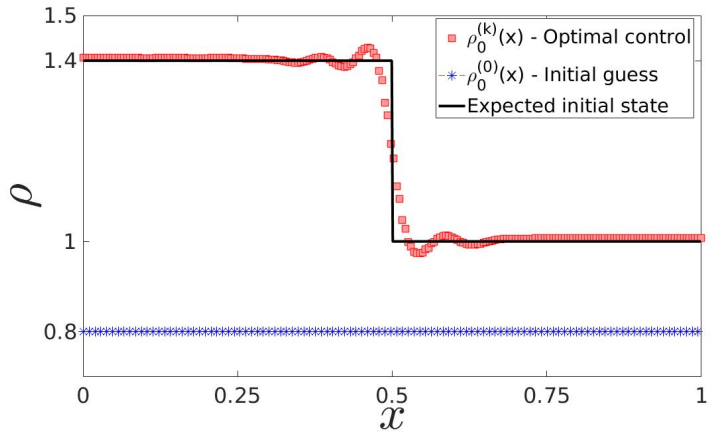
- Isothermal Gas Flow Initial data:

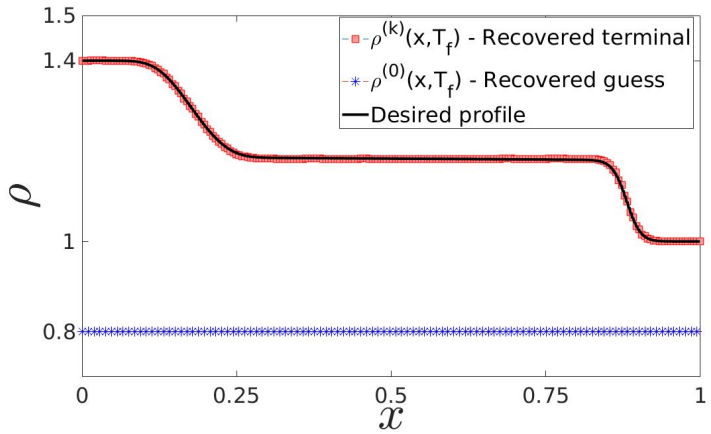
$$[\rho(x,0), u(x,0)] = \begin{cases} [1.4,\ 0], & \text{if } x \leq \frac{1}{2}, \\ [1,0], & \text{if } x > \frac{1}{2} \end{cases}$$

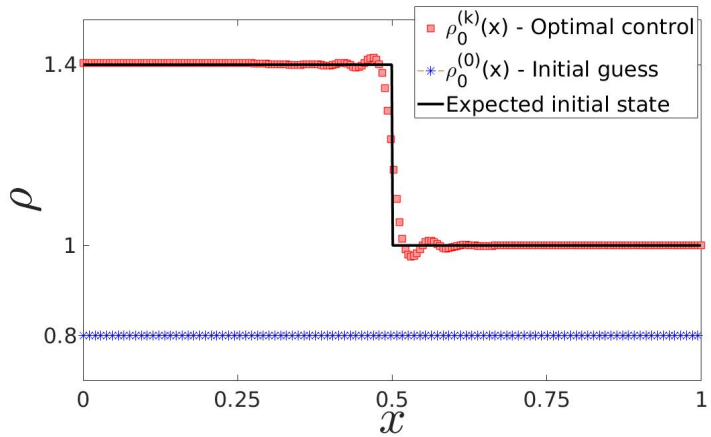
- $x \in [0,1]$, uniform grid $\triangle x = \frac{1}{640}$.
- Adjoint methods, tolerance value 8×10^{-8} as the stopping criteria.

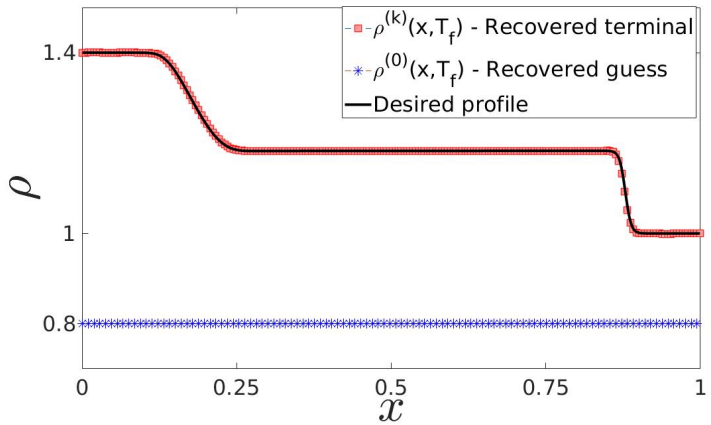
[1] B. Mahlobo

Non-Linear Programming Approach
Density LxF, Rusanov

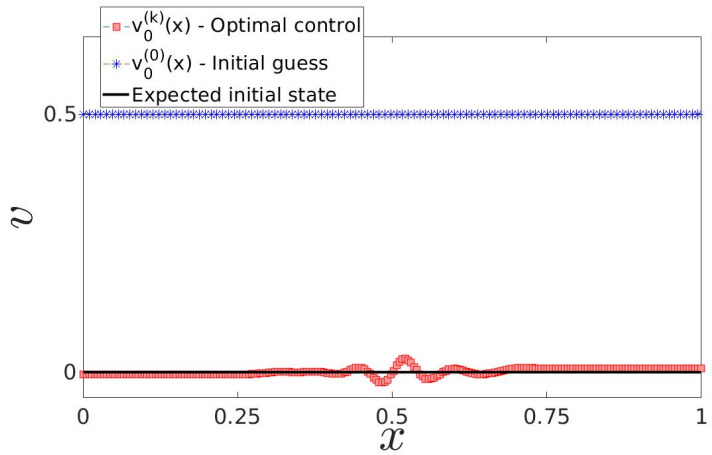


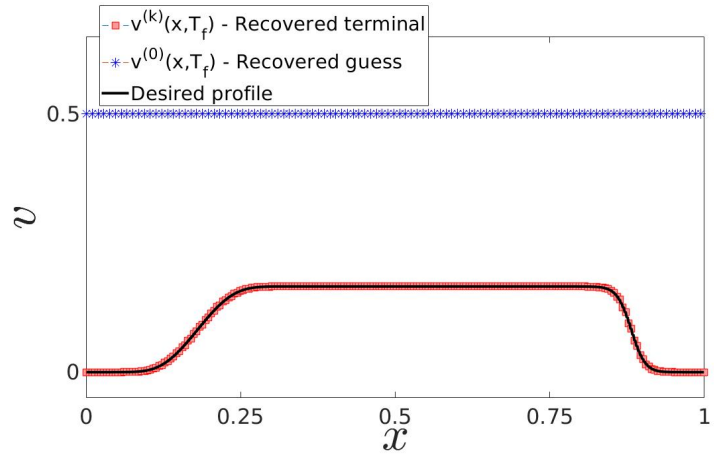


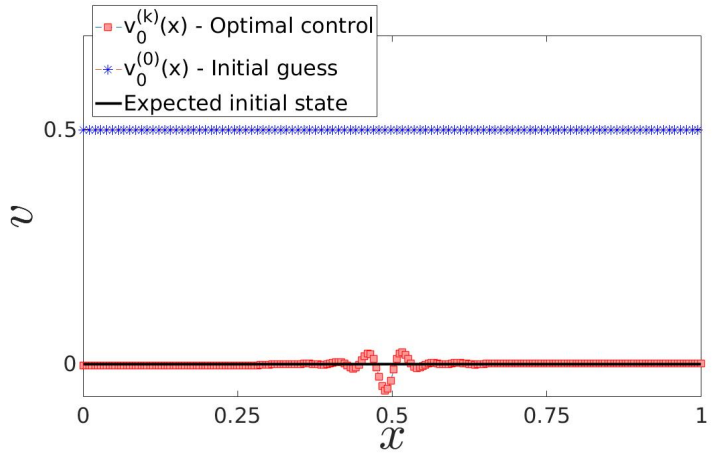


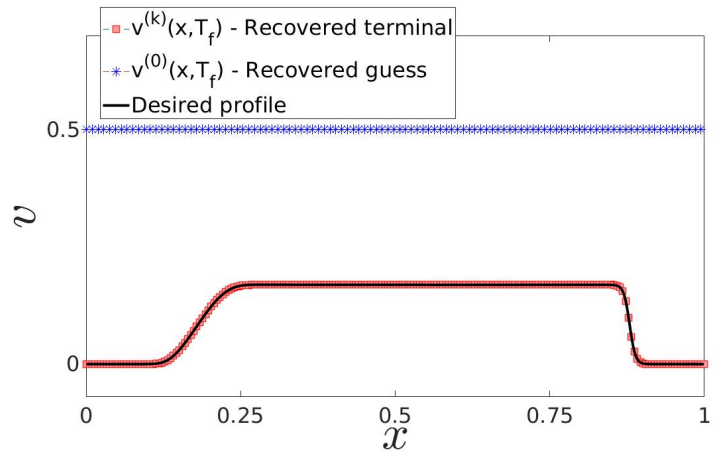


Non-Linear Programming Approach
Velocity LxF, Rusanov









Compressor Optimisation



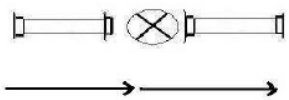
- Cost Functional: $\mathcal{J} = \int_0^T \int_{x_0}^{x_1} \|p(\rho_2)(x,\tau) - \bar{p}\| \, dx d\tau$
- In each pipe:

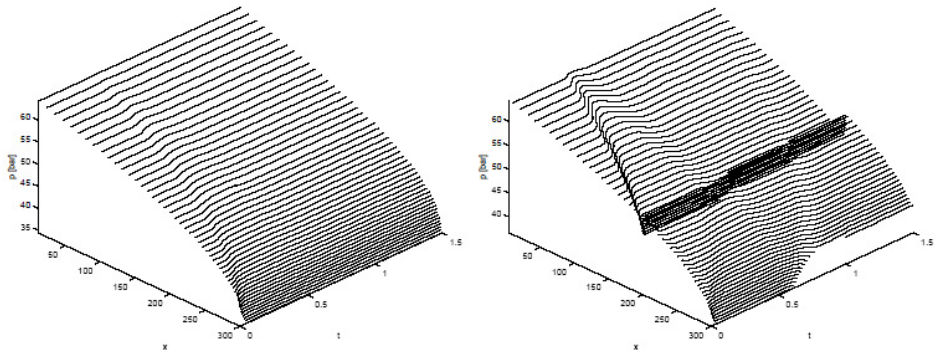
$$\partial_t \begin{pmatrix} \rho^{(e)} \\ \rho^{(e)} u^{(e)} \end{pmatrix} + \partial_x \begin{pmatrix} \rho^{(e)} u^{(e)} \\ p(\rho^{(e)}) + (\rho^{(e)} u^{(e)})^2 \end{pmatrix} = \begin{pmatrix} 0 \\ s(x, \rho^{(e)}, u^{(e)}, g, f_g) \end{pmatrix}$$

- Coupling conditions: $\Psi = \begin{pmatrix} q_1 - q_2 \\ q_2((p(\rho_2)/p(\rho_1))^{\kappa} - 1) - u(t) \end{pmatrix}$

Compressor Optimisation [2]

Idle and active compressor between two pipes with friction and a desired pressure of 42 bar at $x = 300[m]$.





- Minimise the mismatch between output pressure and demand target pressure over a fixed period of time. Stochastic demand?
- Problem is well-posed and the existence of an optimal control TV(u) small proven.

[2] Herty, ...

Flow Stabilisation

Control Goal

Consider

$$\partial_t Y + \partial_x f(Y) + S(Y, x) = 0$$

OR

$$\partial_t Y + F(Y)\partial_x Y + S(Y, x) = 0.$$

- Stabilise a target state Y^* - a desired state, an ideal state, an operating point.
- In river regulations:
  - Y^* may represent a state with a desired and fixed water level for safe navigation, prevent flooding of riverside cities;
- In traffic flow:
  - Y^* may represent uniform flow on a highway, instead of stop-and-go.

Control Goal

Under reasonable regularity assumptions of F and S, change variables

$$\partial_t u + A(u,x)\partial_x u + B(u,x) = 0.$$

- u represents perturbation between Y and Y^*;
- i.e. target state $u \equiv 0$.
- Assuming we have a control on the system: find a boundary feedback law such that perturbations $u \to 0$, if possible, exponentially quickly.
- $$\begin{pmatrix} u_+(0,t) \\ u_-(l,t) \end{pmatrix} = K \begin{pmatrix} u_+(l,t) \\ u_-(0,t) \end{pmatrix}$$

  control is in the boundary conditions where K is the feedback law.

Illustration: 2×2

Conservation Law in Quasilinear Form:

$$\partial_t \begin{pmatrix} u_+ \\ u_- \end{pmatrix} + \begin{pmatrix} c_1(u) & 0 \\ 0 & -c_2(u) \end{pmatrix} \partial_x \begin{pmatrix} u_+ \\ u_- \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$-c_2(u) < 0 < c_1(u).$$

Boundary Conditions: Given as:

$$u_+(0,t) = \kappa_1 u_-(0,t);$$
$$u_-(1,t) = \kappa_2 u_+(1,t);$$

i.e.

$$\begin{pmatrix} u_+(0,t) \\ u_-(1,t) \end{pmatrix} = \begin{pmatrix} 0 & \kappa_1 \\ \kappa_2 & 0 \end{pmatrix} \begin{pmatrix} u_+(1,t) \\ u_-(0,t) \end{pmatrix}.$$

The main tool



Lyapunov Functions

Alexandr Mihailovich Lyapunov (1857 - 1918)

If the total energy is dissipated, then the system must be stable.

In practice ...





- Aim: Design control action at the boundary of domain in order to attain a stable equilibrium status.[3]

[3] Coron2008, Coron2007, Gugat2002, ...

Background

- Existence and Uniqueness Results: linear hyperbolic systems of balance laws under an initial condition, boundary conditions and initial-boundary compatibility conditions[4].
- Stabilisation problems with boundary controls (also called boundary feedbacks or boundary damping) of such systems have been an active research field. [5]
- Studies mainly focus on linear and non-linear systems in $\mathcal{L}^2-$norm and H^2-norm, respectively, in the sense of exponential stability.
- A strict Lyapunov function has played a central role in the investigation of conditions for stability.

[4] Bastin, Coron, Prieur

[5] Bastin, Coron, Diagne, Krstic, Gugat, Herty, Hayat, etc.

Problem Statement

- Analyse numerical feedback boundary stability of linear hyperbolic systems with boundary disturbance [6].
  - Construct a discrete $\mathcal{L}^2-$Lyapunov function.
  - Investigate conditions for exponential stability of discretised hyperbolic systems.
  - Investigate the decay of the discrete $\mathcal{L}^2-$Lyapunov function
  - Compare numerical computations with analytical stability results.

[6] Banda, Goettlich, Herty, Schillen, Dick, Gerster, Weldegiyorgis

Exponential Stability

Definition

We say the Cauchy system is exponentially stable for the $\mathcal{L}^2$ norm with decay rate γ and gain C if there exists constants $\delta > 0$, $\gamma > 0$, and $C > 0$ such that for any $T > 0$ and $u_0 \in \mathcal{L}^2(0, l)$ such that $\|u_0\|_{\mathcal{L}^2} \leq \delta$, the Cauchy problem with initial condition $u(0, \cdot) = u_0$ has a unique solution $u \in C^0([0, T], \mathcal{L}^2(0, l))$ and

$$\|u(\cdot, t)\|_{\mathcal{L}^2} \leq Ce^{-\gamma t}\|u_0\|_{\mathcal{L}^2}.$$

Moreover if $\delta = +\infty$, then the system is said to be globally exponentially stable.

Lyapunov function

Consider the Lyapunov function candidate:

$$\mathcal{V}(u) = \int_0^l (J(x)u(x,t))^T J(x)u(x,t)\, dx,$$

where $J = \text{diag}(J_1, \ldots, J_n) \in C^1([0,l], D_n^+(\mathbb{R}^n))$, where D_n^+ is the space of diagonal matrices with positive coefficients.

- The function $\mathcal{V}$ is well defined on $\mathcal{L}^2(0,l)$ and equivalent to $\|u(\cdot,t)\|_{\mathcal{L}^2}^2$:

$$\|u(\cdot,t)\|_{\mathcal{L}^2}^2 \|J^{-1}\|_{\mathcal{L}^\infty}^{-2} \leq \mathcal{V}(u) \leq \|u(\cdot,t)\|_{\mathcal{L}^2}^2 \|J^{-1}\|_{\mathcal{L}^\infty}^2.$$

- $J(x) = \text{diag}(q_i e^{-\mu s_i x})$ where $s_i = 1$ if $\Lambda_i > 0$ and $s_i = -1$ if $\Lambda_i < 0$ and q_i and μ are positive constants to be chosen.
- Show $\mathcal{V}$ decreases exponentially quickly along u.
- Functions of type $\mathcal{V}$ are referred to as basic quadratic Lyapunov funtion or basic Lyapunov function for the $\mathcal{L}^2$-norm.

Input-to-State stabilisation (ISS)

- System could be subject to unknown disturbances such as imprecisions of the model.
- Consider:

$$\partial_t u + A(u,x)\partial_x u + B(u,x) + d_1(x,t) = 0;$$
$$\begin{pmatrix} u_+(0,t) \\ u_-(l,t) \end{pmatrix} = K \begin{pmatrix} u_+(l,t) \\ u_-(0,t) \end{pmatrix} + d_2(t)$$

where $d_1(x,t)$ are internal disturbances, $d_2(t)$ are boundary disturbances.

Input-to-State stabilisation (ISS)

Aim

- No steady-state anymore to the system, sometimes no hope of stabilising any target state - unknown disturbances!
- Alternative: what is the error we make because of the disturbances when trying to stabilise a steady-state?
- i.e. how robust is the exponential stability with the external disturbances?

Input-to-State stabilisation (ISS)

Remark:

- Show a generic estimate of the form:

$$\|u(\cdot,t)\|_X \leq C_1 e^{-\gamma t}\|u_0\|_X + C_2(\|d_1\|_{X_t \times X} + \|d_2\|_{X_t})$$

  where $\|\cdot\|_X$ is the norm considered over space, $\|\cdot\|_{X_t}$ is the norm considered over time $[0,t]$, $\|\cdot\|_{X_t \times X}$ is the norm over time and space.
- If such an estimate holds: system is exponentially Input-to-State Stable (ISS).
- C_1 and C_2 are the ISS gains.
- If $d_1 \equiv d_2 \equiv 0$ i.e. no disturbances, the definition of exponentially ISS coincides with the definition of exponential stability.

Initial Boundary Value Problem (IBVP)[7]

Consider a $k \times k$ linear hyperbolic system of balance laws:

$$\partial_t u(x,t) + \Lambda(x)\partial_x u(x,t) + \Pi(x)u(x,t) = 0, \quad (x,t) \in [0,l] \times [0,+\infty), \tag{1}$$

- Initial condition:

$$u(x,0) = u_0(x),\ x \in (0,l), \tag{2}$$

  $u_0 : (0,l) \to \mathbb{R}^k$ and
- Linear feedback boundary conditions with disturbances

$$\begin{bmatrix} u^+(0,t) \\ u^-(l,t) \end{bmatrix} = K \begin{bmatrix} u^+(l,t) \\ u^-(0,t) \end{bmatrix} + Mb(t),\ t \in [0,+\infty), \tag{3}$$

where $K = \begin{bmatrix} 0 & K^- \\ K^+ & 0 \end{bmatrix}$ - a constant matrix.

[7] Weldegiyorgis, B

Notion of ISS

- The steady-state $u \equiv 0$ of the IBVP is ISS in $\mathcal{L}^2-$norm with respect to disturbance function b if

$$\|u(\cdot,t)\|^2_{\mathcal{L}^2((0,l);\mathbb{R}^k)} \leq C_1 e^{-\eta t}\|u_0\|^2_{\mathcal{L}^2((0,l);\mathbb{R}^k)} + \frac{C_2}{\eta}\left(1+\frac{1}{\xi}\right)\sup_{s\in[0,t]}\left(|b(s)|^2\right)$$

  $t \in [0,+\infty)$.
- For any continuously differentiable positive definite diagonal matrix $P(x) = \text{diag}\{p_1(x),\ldots,p_k(x)\}$, $x \in [0,l]$, an $\mathcal{L}^2-$function defined by

$$\mathcal{V}(u(\cdot,t)) = \int_0^l u^\top P(x) u \, dx, \;\; t \in [0,+\infty), \tag{4}$$

  is said to be an ISS-Lyapunov function for the IBVP if

$$\frac{d\mathcal{V}(u(\cdot,t))}{dt} \leq -\eta\mathcal{V}(u(\cdot,t)) + \nu\left(1+\frac{1}{\xi}\right)\sup_{s\in[0,t]}\left(|b(s)|^2\right).$$

Theoretical stability result

Theorem (Stability)

Under appropriate assumptions including:

- *Internal Conditions:*

$$-\Lambda(x)P'(x) - \Lambda'(x)P(x) + \Pi^{\top}(x)P(x) + P(x)\Pi(x),$$

is positive definite for all $x \in [0, l]$

The stability result

Theorem (Stability Continued)

and

- *Boundary Conditions: the matrix*

$$\begin{bmatrix} \Lambda^+(I)P^+(I) & 0 \\ 0 & \Lambda^-(0)P^-(0) \end{bmatrix} - (1+\xi)\, K^\top \begin{bmatrix} \Lambda^+(0)P^+(0) & 0 \\ 0 & \Lambda^-(I)P^-(I) \end{bmatrix} K,$$

  is positive semi-definite.

The stability result continued

Theorem (Stability continued)

Moreover, let ν be the largest eigenvalue of the matrix

$$M^\top \begin{bmatrix} \Lambda^+(0)P^+(0) & 0 \\ 0 & \Lambda^-(l)P^-(l) \end{bmatrix} M.$$

Then the $\mathcal{L}^2-$ function is an ISS-Lyapunov function for the IBVP. Moreover, the steady-state $u(x,t) \equiv 0$ of the system is ISS in $\mathcal{L}^2-$norm with respect to the disturbance function b.

Numerical discretisation

- The approximate cell average of u over the j^{th} cell at time t^n $(n = 0, \ldots, N)$:

$$u_j^n = \frac{1}{\Delta x} \int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} u(x, t^n)\, dx,\ j = 0, \ldots, J-1,$$

- For a smooth solution $u(x, t)$, the integral approximation:

$$\int_0^l u(x, t^n) dx = \Delta x \sum_{j=0}^{J-1} u_j^n,\ n = 0, \ldots, N-1.$$

- The solution $u(x_j, t^n)$ is approximated by u_j^n: use Euler in time and Upwind in space.

Theoretical Stability results

Theorem (Stability)

Under appropriate assumptions and equivalent discrete LMI as above. Then the discrete $\mathcal{L}^2-$function is a discrete ISS-Lyapunov function for the discretised IBVP and parameters ξ, ν. Moreover, the steady-state $u_j^n \equiv 0,\ j = 0, \ldots, J-1,\ n = 0, \ldots, N-1$ of the discretised IBVP is discrete ISS in $\mathcal{L}^2-$norm with respect to discrete disturbance function $b^n,\ n = 1, \ldots, N$.

Example 1: Saint-Venant equations

- Initial condition

$$H(x,0) = H_0(x), \quad V(x,0) = V_0(x), \quad x \in (0,l),$$

- Boundary conditions with disturbances

$$V(0,t) = k_0 H(0,t) + b_1(t), \quad V(l,t) = k_l H(l,t) + b_2(t), \quad t \in [0,\infty),$$

where k_0, k_l are boundary control parameters, and b_1, b_2 are disturbance functions.

- Assume $H(x,t) > 0$, $V(x,t) > 0$ for all $x \in [0,l]$ and $t \in [0,+\infty)$.
- Obtain a non-uniform steady-state solution, $H^*(x)$, $V^*(x)$.

Example 1: Saint-Venant equations [8]

- non-uniform steady-state solution $(H^*(x), V^*(x))$ can be obtained by solving an ODE
- with $H^*(0) = 2m$, $V^*(0) = 0.5m/s$, $C_f = 0.002$, $g = 9.81m/s^2$ and $S_b = \frac{2C_f V^{*2}(0)}{gH^*(0)}$.
- Initial condition $(H(x,0), V(x,0)) = (2.5, 4\sin(\pi x))$ for $x \in [0,1]$.
- The rate of the boundary disturbance functions taken as $b_1(t) = -b_2(t) = 0.01\sin(\pi t),\ t > 0$.

[8] Hayat, 2019

Example 1: Saint-Venant equations

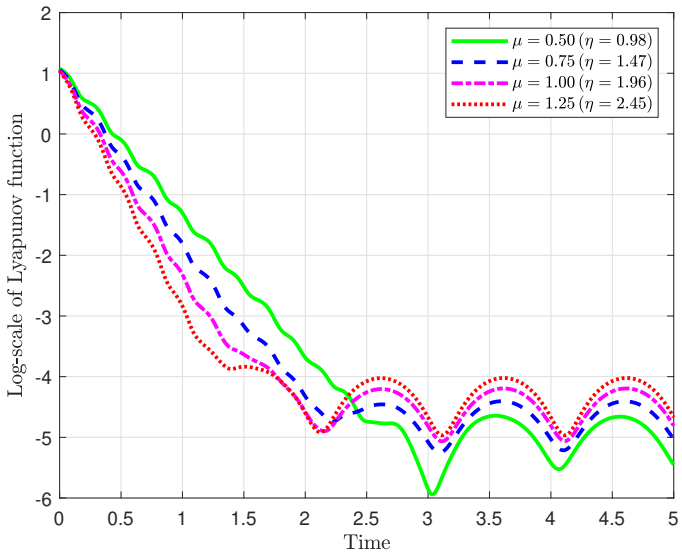


Figure: Parameters are $p_1 = 1, p_2 = 1, \xi = 1, \kappa_{12} = 0.6, \kappa_{21} = 0.8\exp(-\mu), m_1 = 1 - \kappa_{12}, m_2 = 1 - \kappa_{21}$ with $I = 1$, $J = 100$ and $T = 5$ under CFL $= 0.75$.

μ increases the rate of decay of the ISS-Lyapunov function in the presence of boundary disturbance increases.

Conclusion

Conclusion

- Extension to networks including coupling conditions.
- Application of high-order schemes versus low-order schemes.
- Efficiency issues, multi-fidelity approaches; Multiscale analysis for networks.
- Integration of other components of networks: compressors, valves, etc. - MIP approaches
- Boundary feedback stabilisation Extensions:
  1. Additive disturbances.
  2. Numerical analysis of Lyapunov functions for semi-linear differential equations.
  3. Analysis of numerical artefacts such as numerical viscosity need to be carefully investigated.

Overview reference



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M. Herty, Hyperbolic partial differential equations in optimization and control, SIAM News, Vol 59(4), 2026.

Dankie, Thank You, Ke a Leboga!

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