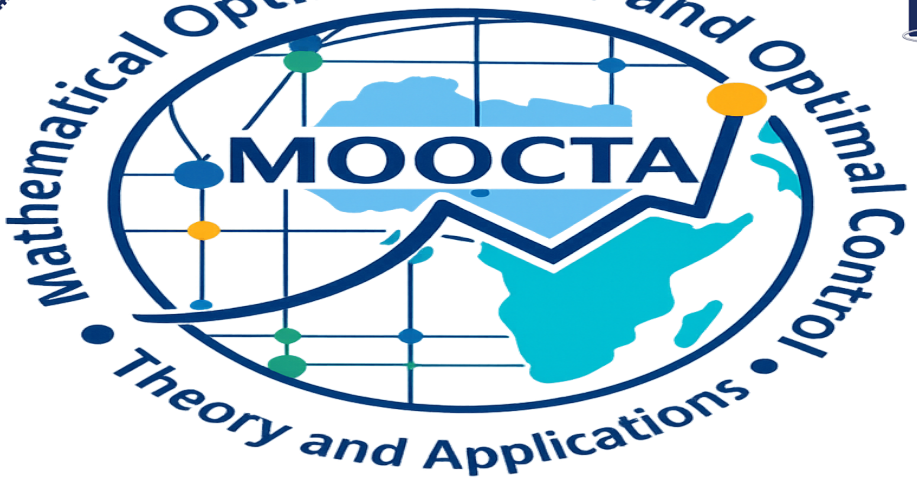

Mathematical Optimization and Optimal Control
MOOCTA
Theory and Applications

Robust Portfolio Optimisation for Uncertain Investment Variables

Supervisors: Prof. Sunday Reju and Prof. Serge Neossi

Ndapandula Peneyambeko Hamupolo

Namibia University of Science and Technology

Department of Mathematics, Statistics and Actuarial Sciences

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Outline

Introduction to Portfolio Optimisation

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References

Introduction and Background

- **Portfolio optimisation** determines how capital is allocated across assets while balancing **return and risk**.
- The foundational framework is the **Mean–Variance model** introduced by **Harry Markowitz (1952)**.
- It uses historical estimates of **expected returns and covariance** to construct portfolios with desirable risk–return characteristics.
- However, these estimates are uncertain, and small estimation errors can lead to **substantial changes in portfolio weights** (Xidonas et al., 2020).

Portfolio Risk, Return and the Markowitz Model

Portfolio Expected Return

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \tag{1}$$

Portfolio Variance

$$\sigma_p^2 = \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} \tag{2}$$

Classical Markowitz Model

$$\left.\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} \\ \text{subject to} \quad & E(r_p) \geq \mu_0, \\ & \sum_{i=1}^{n} w_i = 1, \qquad w_i \geq 0, \quad i = 1, \ldots, n. \end{aligned}\right\} \tag{3}$$

- $\mathbf{w}$: portfolio-weight vector; $\mathbf{\Sigma}$: covariance matrix;
- $E(r_p)$: expected portfolio return; μ_0: required minimum portfolio return.

Efficient Frontier

Efficient Frontier

- The **efficient frontier** represents the best attainable combinations of **portfolio risk and return**.
- Each portfolio on the frontier provides the **minimum possible risk for a given expected return**.
- It therefore shows the trade-off between **risk and expected return**.

Statement of the Problem

Research Problem

Although the classical Markowitz model identifies efficient risk–return portfolios, these portfolios depend on estimated market parameters that are subject to uncertainty.

- **Estimation uncertainty** in expected returns and covariance estimates can significantly affect optimal portfolio weights (Maillet et al., 2015; Patel & Chakraborty, 2017).
- Consequently, a portfolio that appears optimal based on historical information may not perform as expected under future market conditions, creating a gap between **in-sample optimality** and **out-of-sample performance**.
- This motivates the investigation of **robust portfolio optimisation models** that account for parameter uncertainty.

Objectives of the Study

Overall Aim

To evaluate the effectiveness of robust portfolio optimisation models in improving portfolio performance under **parameter uncertainty**.

Specific Objectives

1. To implement and analyse the **classical Markowitz model and its robust counterparts**, including Box, Ellipsoidal, robust Mean–Conditional Value-at-Risk (Mean–CVaR), and Distributionally Robust CVaR models using historical financial data.
2. To compare the implemented models in terms of **portfolio risk, return, risk-adjusted performance, and portfolio allocations** under parameter uncertainty, using the classical Markowitz model as the benchmark.
3. To evaluate the **out-of-sample performance** of the implemented models through backtesting using portfolio performance and risk measures

Literature Review: Evolution of Portfolio Optimisation

1. **Mean–Variance Optimisation**
   Balances expected return and portfolio risk (Markowitz, 1952).
2. **Stochastic Optimisation**
   Models uncertain returns using probability distributions(Kall & Wallace, 1994).
3. **Robust Optimisation** Protects portfolio decisions against parameter uncertainty (Ben-Tal & Nemirovski, 1999; Goldfarb & Iyengar, 2003).
4. **Structured Uncertainty Sets**
   Box and ellipsoidal uncertainty provide alternative representations of parameter uncertainty (Goldfarb & Iyengar, 2003).
5. **Robust CVaR**
   Extends robustness to downside-risk management (Huang et al., 2010).
6. **Distributionally Robust Optimisation**
   Accounts for uncertainty in the underlying return distribution (Ghahtarani et al., 2022; Pflug & Pohl, 2018).

Methodology

- Historical daily price data for **30 selected U.S. equities** are obtained from the **Bloomberg Terminal** for the period **1 January 2018–31 December 2025**.
- Prices are converted into **daily logarithmic returns**:

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right), \tag{4}$$

where $P_{i,t}$ denotes the price of asset i at time t.

- The resulting return series are used as inputs for the **selected portfolio optimisation models**.
- The models use consistent **investment constraints**,including full investment and no short selling, with model-specific risk and uncertainty formulations.

Classical Markowitz Mean–Variance Model

- The classical Markowitz model provides the **benchmark** for evaluating the robust mean–variance models.
- Portfolio risk is measured by the variance of portfolio returns.

$$\begin{array}{lllll} \min\limits_{\mathbf{w}} & \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} & & & \\ \text{s.t.} & \mathbf{w}^T\hat{\boldsymbol{\mu}} \geq \mu_0, & \mathbf{1}^T\mathbf{w} = 1, & \mathbf{w} \geq 0. \end{array} \tag{5}$$

- $\mathbf{w}$: portfolio-weight vector; $\hat{\boldsymbol{\mu}}$: estimated expected-return vector;
- $\boldsymbol{\Sigma}$: covariance matrix; μ_0: required minimum portfolio return.

Key Idea

This study focuses on the **minimum-variance formulation** of the Markowitz model, which minimises portfolio risk while satisfying the required return, full-investment, and no-short-selling constraints.

Robust Markowitz: General Formulation

The robust Markowitz model incorporates uncertainty in the estimated expected returns through an uncertainty set $\mathcal{U}$.

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \\ \text{s.t.} \quad & \min_{\hat{\boldsymbol{\mu}} \in \mathcal{U}} \mathbf{w}^T \hat{\boldsymbol{\mu}} \geq \mu_0, \\ & \mathbf{1}^T \mathbf{w} = 1, \qquad \mathbf{w} \geq 0. \end{aligned} \tag{6}$$

where

- $\hat{\boldsymbol{\mu}}$ is the estimated expected-return vector;
- $\mathcal{U}$ is the uncertainty set; and
- μ_0 is the required minimum portfolio return.

Two uncertainty sets are considered:

Box uncertainty and Ellipsoidal uncertainty.

Robust Markowitz: Box Uncertainty

The Box uncertainty set allows each expected return to deviate independently from its estimate:

$$\boxed{\mathcal{U}_B(\hat{\boldsymbol{\mu}}) = \left\{ \boldsymbol{\mu} \in \mathbb{R}^N \;\middle|\; |\mu_i - \hat{\mu}_i| \leq \delta_i,\ i = 1, \ldots, N \right\}.} \tag{7}$$

Equivalently, each uncertain expected return satisfies

$$\boxed{\hat{\mu}_i - \delta_i \leq \mu_i \leq \hat{\mu}_i + \delta_i, \qquad i = 1, \ldots, N.} \tag{8}$$

For the long-only portfolio, $w_i \geq 0$, the worst-case portfolio return is obtained by minimizing $\mathbf{w}^T \boldsymbol{\mu}$ over the Box uncertainty set. Since all portfolio weights are non-negative, the minimum occurs when each asset takes its lowest allowable expected return:

$$\mu_i = \hat{\mu}_i - \delta_i.$$

Therefore,

$$\boxed{\min_{\boldsymbol{\mu} \in \mathcal{U}_B} \mathbf{w}^T \boldsymbol{\mu} = \mathbf{w}^T (\hat{\boldsymbol{\mu}} - \boldsymbol{\delta})} \tag{9}$$

Thus, $\mathbf{w}^T(\hat{\boldsymbol{\mu}} - \boldsymbol{\delta})$ represents the worst-case expected portfolio return allowed by the Box uncertainty set.

Box-Robust Markowitz: Estimation and Counterpart

The uncertainty deviation is calibrated following Fabozzi et al. (2007), using a conventional 95% confidence-level scaling:

$$\boxed{\delta_i = 1.96 \frac{\sigma_i}{\sqrt{T}}} \tag{10}$$

This is treated as a benchmark calibration of parameter uncertainty rather than a strict distributional assumption on returns. A sensitivity analysis will assess the robustness of the portfolio results to alternative uncertainty levels.

Hence, the numerical **Box-Robust Markowitz model**, solved using the MATLAB `quadprog` solver, is given by:

$$\boxed{\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^T (\hat{\boldsymbol{\mu}} - \boldsymbol{\delta}) \geq \mu_0, \\ & \mathbf{1}^T \mathbf{w} = 1, \qquad \mathbf{w} \geq 0. \end{aligned}} \tag{11}$$

Robust Markowitz: Ellipsoidal Uncertainty

Unlike the Box set, the ellipsoidal set accounts for the **covariance structure of estimation errors**.

$$\boxed{\mathcal{U}_E(\hat{\boldsymbol{\mu}}) = \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}})^T \boldsymbol{\Sigma}_\mu^{-1} (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}) \leq \delta^2 \right\}.} \tag{12}$$

For a **95% joint confidence region**, the chi-square distribution is used because the uncertainty is considered jointly across the N expected-return estimates:

$$\boxed{\delta = \sqrt{\chi^2_{N,\,0.95}} = \sqrt{\chi^2_{30,\,0.95}}} \tag{13}$$

where $\boldsymbol{\Sigma}_\mu$ represents the covariance structure of the estimation errors.

Thus, the ellipsoidal set defines a **joint confidence region** for the vector of expected returns.

Key Idea

The ellipsoidal set captures **joint estimation uncertainty** through the covariance structure of expected returns.

Ellipsoidal-Robust Markowitz: Counterpart

The worst-case expected portfolio return over the ellipsoidal uncertainty set is

$$\min_{\boldsymbol{\mu}\in\mathcal{U}_E} \mathbf{w}^T\boldsymbol{\mu} = \mathbf{w}^T\hat{\boldsymbol{\mu}} - \delta\sqrt{\mathbf{w}^T\boldsymbol{\Sigma}_\mu\mathbf{w}}. \tag{14}$$

Therefore, the robust target-return constraint becomes

$$\mathbf{w}^T\hat{\boldsymbol{\mu}} - \delta\sqrt{\mathbf{w}^T\boldsymbol{\Sigma}_\mu\mathbf{w}} \geq \mu_0. \tag{15}$$

Hence, the numerical **Ellipsoidal-Robust Markowitz model** is

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^T\hat{\boldsymbol{\mu}} - \delta\sqrt{\mathbf{w}^T\boldsymbol{\Sigma}_\mu\mathbf{w}} \geq \mu_0, \\ & \mathbf{1}^T\mathbf{w} = 1, \qquad \mathbf{w} \geq 0. \end{aligned} \tag{16}$$

Classical Mean–Conditional Value at Risk Model

At a **95% confidence level**, CVaR measures the **average loss in the worst 5% of portfolio outcomes**.

Portfolio loss is defined as the negative of portfolio return:

$$\boxed{L(\mathbf{w}, \mathbf{r}_t) = -\mathbf{w}^T \mathbf{r}_t} \tag{17}$$

where $\mathbf{r}_t$ is the vector of asset returns at time t.

Let γ denote the **VaR threshold**. The excess loss is

$$\boxed{u_t = \max\left\{-\mathbf{w}^T \mathbf{r}_t - \gamma,\ 0\right\}} \tag{18}$$

Key Idea

Only losses exceeding the VaR threshold contribute to the **tail loss**.

Classical Mean–CVaR: Empirical Formulation

For the T historical daily return observations, the empirical CVaR at the **95% confidence level** is

$$\boxed{\mathrm{CVaR}_{0.95} = \gamma + \frac{1}{(1-0.95)\,T} \sum_{t=1}^{T} u_t} \tag{19}$$

The classical Mean–CVaR optimisation problem is

$$\boxed{\begin{aligned} \min_{\mathbf{w},\gamma,\mathbf{u}} \quad & \gamma + \frac{1}{(1-0.95)\,T} \sum_{t=1}^{T} u_t \\ \text{s.t.} \quad & u_t \geq -\mathbf{w}^T \mathbf{r}_t - \gamma, \qquad u_t \geq 0, \\ & \mathbf{w}^T \hat{\boldsymbol{\mu}} \geq \mu_0, \\ & \mathbf{1}^T \mathbf{w} = 1, \qquad \mathbf{w} \geq 0. \end{aligned}} \tag{20}$$

Robust Mean–CVaR with Box Uncertainty

The robust Mean–CVaR counterpart is

$$\begin{aligned} \min_{\mathbf{w},\gamma,\mathbf{u}} \quad & \gamma + \frac{1}{(1-0.95)T}\sum_{t=1}^{T} u_t \\ \text{s.t.} \quad & \mathbf{w}^T(\hat{\boldsymbol{\mu}} - \boldsymbol{\delta}) \geq \mu_0, \\ & u_t \geq -\mathbf{w}^T\mathbf{r}_t - \gamma, \qquad u_t \geq 0, \\ & \mathbf{1}^T\mathbf{w} = 1, \qquad \mathbf{w} \geq 0. \end{aligned} \tag{21}$$

Key Idea

The robust Mean–CVaR model combines **worst-case return protection** with **tail-risk minimisation**.

Wasserstein Uncertainty Set

The historical Bloomberg returns give us an **empirical distribution**. However, the true future distribution may be different.

The Wasserstein DRO model therefore considers **nearby plausible distributions** within an uncertainty region:

$$\boxed{\mathcal{D}_\epsilon = \left\{ P : W(P, \hat{P}_T) \leq \epsilon \right\}} \tag{22}$$

where $W(P, \hat{P}_T)$ measures the distance between a possible distribution P and the empirical distribution $\hat{P}_T$.

The parameter $\epsilon > 0$ controls how much the true distribution is allowed to differ from the historical distribution.

Key Idea

DRO does not rely only on the historical distribution. It considers **nearby plausible distributions** to account for distributional uncertainty.

Distributionally Robust Mean–CVaR Model

The model protects the portfolio against the **worst-case distribution** within the Wasserstein uncertainty set:

$$\begin{aligned} \min_{\mathbf{w},\gamma} \quad & \sup_{P\in\mathcal{D}_\epsilon} \left\{ \gamma + \frac{1}{1-0.95}\mathbb{E}_P\left[\left(-\mathbf{w}^T\mathbf{r}-\gamma\right)^+\right]\right\} \\ \text{s.t.} \quad & \inf_{P\in\mathcal{D}_\epsilon} \mathbb{E}_P\left[\mathbf{w}^T\mathbf{r}\right] \geq \mu_0, \\ & \mathbf{1}^T\mathbf{w} = 1, \qquad \mathbf{w} \geq 0. \end{aligned} \tag{23}$$

Key Idea

The model minimises **worst-case CVaR** while ensuring that the portfolio achieves the required return under distributional uncertainty.

Analysis of the models

The analysis considers **30 selected U.S. equities** representing different sectors of the U.S. equity market. Bloomberg ticker symbols were used to retrieve the historical market data.

Table 1: Selected U.S. Equities

No.	Company	Ticker
1	Apple Inc.	AAPL
2	Microsoft Corp.	MSFT
3	NVIDIA Corp.	NVDA
4	Alphabet Inc.	GOOGL
5	Broadcom Inc.	AVGO
6	Amazon.com Inc.	AMZN
7	Tesla Inc.	TSLA
8	Walt Disney Co.	DIS
9	McDonald's Corp.	MCD
10	Johnson & Johnson	JNJ
11	Eli Lilly and Co.	LLY
12	AbbVie Inc.	ABBV
13	Merck & Co., Inc.	MRK
14	JPMorgan Chase & Co.	JPM
15	Bank of America Corp.	BAC
16	Goldman Sachs Group	GS
17	Visa Inc.	V
18	Mastercard Inc.	MA
19	Exxon Mobil Corp.	XOM
20	Chevron Corp.	CVX
21	Caterpillar Inc.	CAT
22	Honeywell International	HON
23	Coca-Cola Co.	KO
24	PepsiCo, Inc.	PEP
25	Procter & Gamble Co.	PG
26	Walmart Inc.	WMT
27	Costco Wholesale Corp.	COST
28	NextEra Energy, Inc.	NEE
29	American Tower Corp.	AMT
30	Linde plc	LIN

Historical Price Data and Study Period

- Daily share-price data were obtained from the **Bloomberg Terminal**.
- The raw dataset for the period contained **2,932 calendar-day observations** per asset.
- The data were examined and cleaned before the empirical analysis.

Study Period

1 January 2018 – 31 December 2025

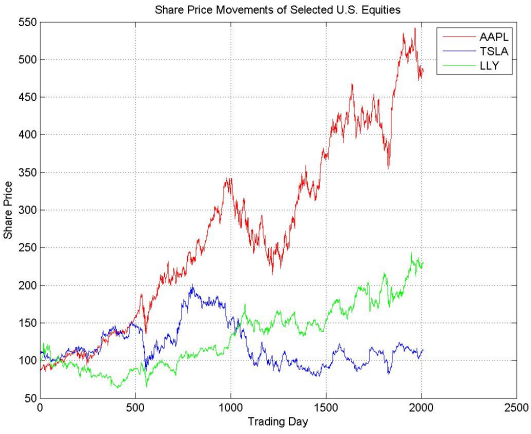


Figure 1: Share-price movements of AAPL, TSLA and LLY.

Data Cleaning and Trading-Day Observations

- U.S. equity markets are closed on **weekends and official market holidays**.
- These non-trading observations were removed from the calendar-day dataset.
- After cleaning, **2,011 trading-day observations** remained for each asset.

Final Dataset

2,011 trading days $\times$ 30 assets

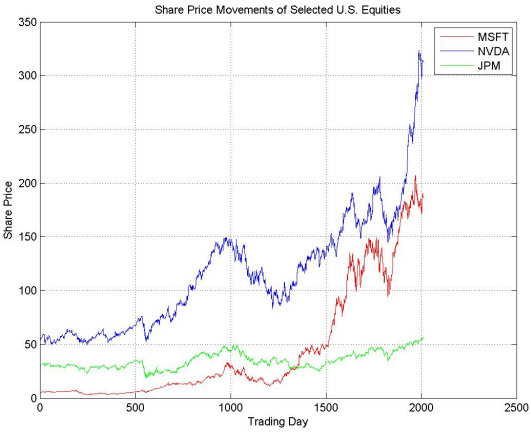


Figure 2: Share-price movements of MSFT, NVDA and JPM.

Calculation of Daily Logarithmic Returns

The cleaned closing prices were converted into **daily logarithmic returns**, which are additive over time and suitable for portfolio optimisation.
Let $P_{j,t}$ denote the closing price of asset j on trading day t.

$$\boxed{r_{j,t} = \ln\left(\frac{P_{j,t}}{P_{j,t-1}}\right)} \qquad (24)$$

where

$$j = 1, \ldots, 30, \qquad t = 2, \ldots, 2,011.$$

Hence, $\boxed{T = 2,010}$ daily log returns are obtained per asset.

The resulting return matrix is

$$\mathbf{R} = \begin{bmatrix} r_{\mathrm{AAPL},1} & r_{\mathrm{MSFT},1} & \cdots & r_{\mathrm{LIN},1} \\ r_{\mathrm{AAPL},2} & r_{\mathrm{MSFT},2} & \cdots & r_{\mathrm{LIN},2} \\ \vdots & \vdots & \ddots & \vdots \\ r_{\mathrm{AAPL},2010} & r_{\mathrm{MSFT},2010} & \cdots & r_{\mathrm{LIN},2010} \end{bmatrix}. \qquad (25)$$

$$\boxed{\mathbf{R} \in \mathbb{R}^{2010 \times 30}}$$

Key Point

The return matrix provides the input data for estimating $\hat{\boldsymbol{\mu}}$ and $\boldsymbol{\Sigma}$ and solving the portfolio optimisation models.

Table 2: Estimated Mean Daily Returns

Asset	Mean	Asset	Mean
AAPL	0.0917%	MSFT	0.0859%
NVDA	0.1802%	GOOGL	0.0877%
AVGO	0.1275%	AMZN	0.0675%
TSLA	0.1516%	DIS	0.000869%
MCD	0.0282%	JNJ	0.0197%
LLY	0.1264%	ABBV	0.0419%
MRK	0.0312%	JPM	0.0544%
BAC	0.0303%	GS	0.0614%
V	0.0557%	MA	0.0659%
XOM	0.0173%	CVX	0.00885%
CAT	0.0644%	HON	0.0170%
KO	0.0213%	PEP	0.00972%
PG	0.0228%	WMT	0.0607%
COST	0.0757%	NEE	0.0362%
AMT	0.0109%	LIN	0.0500%

Table 3: Estimated Covariance Matrix

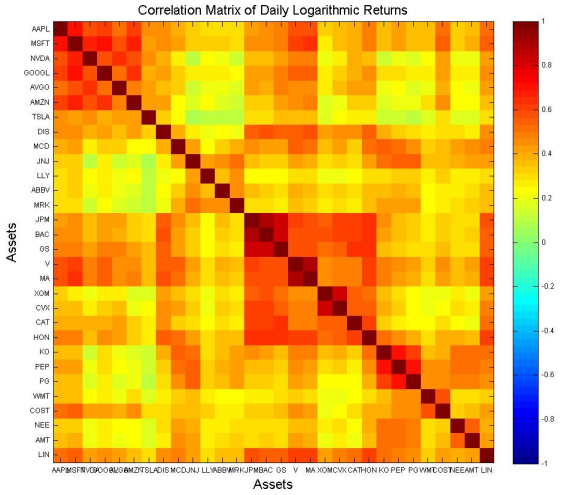
	AAPL	MSFT	NVDA	GOOGL	AVGO	···	NEE	AMT	LIN
AAPL	.000375	.000243	.000359	.000236	.000265	···	.000110	.000116	.000149
MSFT	.000243	.000319	.000383	.000248	.000263	···	.000104	.000117	.000147
NVDA	.000359	.000383	.001042	.000365	.000541	···	.000114	.000118	.000204
GOOGL	.000236	.000248	.000365	.000381	.000254	···	.000091	.000092	.000135
AVGO	.000265	.000263	.000541	.000254	.000678	···	.000101	.000092	.000163
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	
NEE	.000110	.000104	.000114	.000091	.000101	···	.000294	.000163	.000105
AMT	.000116	.000117	.000118	.000092	.000092	···	.000163	.000300	.000110
LIN	.000149	.000147	.000204	.000135	.000163	···	.000105	.000110	.000233

Correlation Structure of Asset Returns

Correlation Analysis

The correlation matrix was computed from the daily logarithmic returns of the 30 selected U.S. equities.

It provides the dependence structure between the asset returns and supports the assessment of portfolio diversification.



Normality Test of Daily Returns

Jarque–Bera Test

- The Jarque–Bera test was applied at the **5% significance level**.
- Normality was **rejected for all 30 assets**.
- The daily logarithmic returns therefore **do not follow a normal distribution**.

Result

Non-normal return distributions

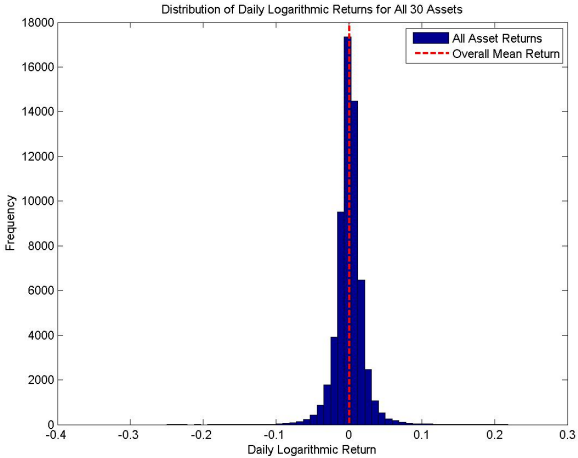


Figure: **Figure 4:** Distribution of daily logarithmic returns for all 30

Classical Markowitz: Optimal Portfolio

Table 5: Optimal Portfolio Weights

Asset	Weight
DIS	74.9994%
NVDA	13.1471%
TSLA	7.0617%
LLY	2.5235%
AVGO	2.2684%
Other 25 assets	**0.0000%**

Interpretation

The portfolio is highly concentrated, with **nearly 75% in DIS** and **25.0006% across four other assets**.

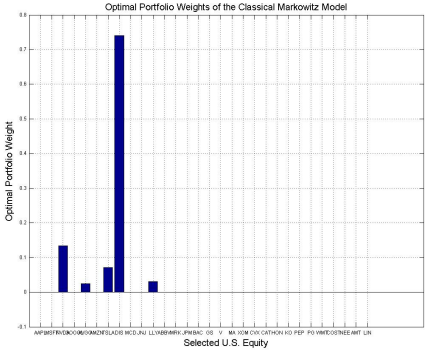


Figure: **Figure 5:** Optimal portfolio weights of the classical Markowitz model.

Classical Markowitz: Performance Measures

Table 6: Performance Measures

Measure	Value
Expected Daily Return	0.042534%
Portfolio Variance	0.033329%
Portfolio Std. Deviation	1.825610%
Daily Risk-Free Rate	0.010050%
Sharpe Ratio	0.017794
Sortino Ratio	0.022460
Omega Ratio	1.070781
Maximum Drawdown	-55.4793%

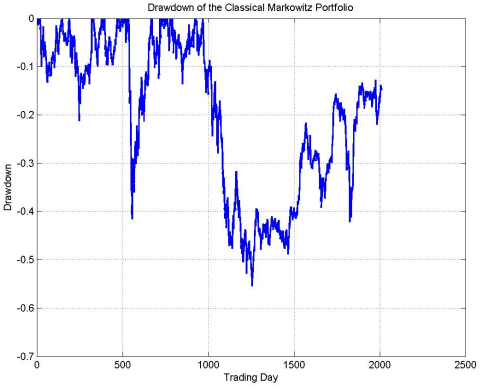


Figure 6: Drawdown of the optimal classical Markowitz portfolio.

Key Interpretation

The portfolio fell by a maximum of **55.48%** from its previous highest value.

Box Uncertainty Set

$$\mathcal{U}_{\mathrm{B}} = \left\{ \boldsymbol{\mu} \in \mathbb{R}^{30} \;\middle|\; |\mu_j - \hat{\mu}_j| \leq \delta_j,\ j = 1, \ldots, 30 \right\} \tag{26}$$

Equivalently,

$$\hat{\mu}_j - \delta_j \leq \mu_j \leq \hat{\mu}_j + \delta_j \tag{27}$$

The uncertainty radius is calculated separately for each asset at a **95% confidence level**:

$$\delta_j = 1.96 \frac{s_j}{\sqrt{T}}, \qquad j = 1, \ldots, n \tag{28}$$

Estimated Mean Return Intervals

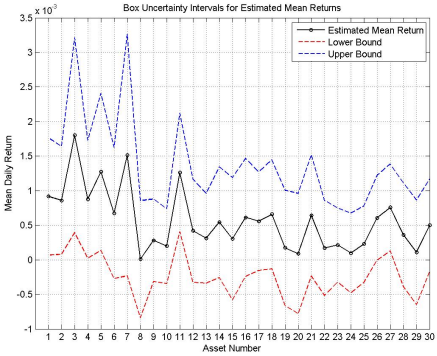


Figure 7: Box uncertainty intervals for the estimated mean daily returns.

Box-Robust Markowitz: Optimal Portfolio

Table 7: Portfolio Solution

Measure	Value
Worst-case Daily Return	0.02442%
Portfolio Variance	0.01376%
Portfolio Std. Deviation	1.17323%

Optimal Portfolio Weights

Table 8:

Asset	Weight	Asset	Weight
JNJ	18.5907%	WMT	16.0239%
KO	15.0135%	MCD	11.4652%
PG	9.4112%	MRK	9.2164%
AMZN	5.6136%	XOM	5.4251%
COST	5.0320%	ABBV	3.4708%
GOOGL	0.3071%	CAT	0.2311%
LLY	0.1994%	Other	0.0000%

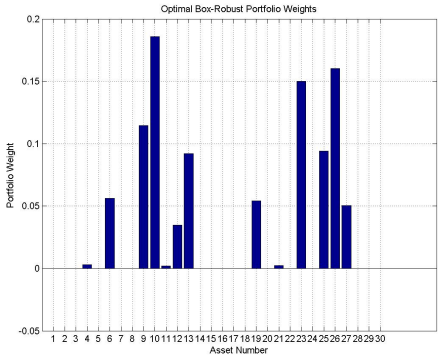


Figure 8: Optimal portfolio weights under the Box-Robust Markowitz model.

Interpretation

Box-Robust Portfolio Performance

Table 9: Box-Robust Portfolio

Performance Measure	Value
Mean daily return	0.0355%
Portfolio variance	0.00875%
Standard deviation	0.9356%
Daily risk-free rate	0.0101%
Sharpe ratio	0.027223
Sortino ratio	0.052837
Omega ratio	1.121405
95% VaR	1.3409%
95% CVaR	2.2099%
Maximum drawdown	26.7811%

- The portfolio achieved a **0.0355%** mean daily return.
- The **Omega ratio of 1.12** indicates gains exceeded losses.

Drawdown

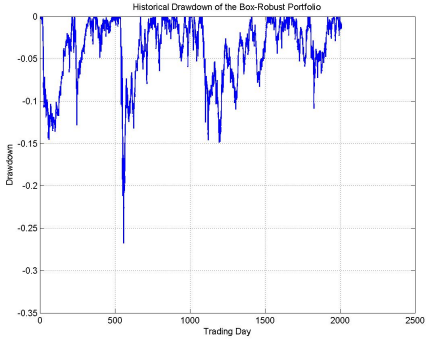


Figure 9: Drawdown of the optimal Box-Robust portfolio.

- Maximum drawdown was **26.78%**.

Ellipsoidal Uncertainty Set

Unlike the Box set, the ellipsoidal set allows the mean returns to vary **jointly**, accounting for their estimation-error correlations.

$$\mathcal{U}_\delta(\hat{\boldsymbol{\mu}}) = \left\{ \boldsymbol{\mu} \in \mathbb{R}^{30} \;\middle|\; (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}})^\top \boldsymbol{\Sigma}_\mu^{-1} (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}) \leq \delta^2 \right. \tag{29}$$

The covariance of the estimation errors is

$$\boldsymbol{\Sigma}_\mu = \frac{1}{T}\boldsymbol{\Sigma}, \qquad T = 2010 \tag{30}$$

At the **95% confidence level**,

$$\delta^2 = \chi^2_{30,0.95} = 43.773, \qquad \delta = 6.616 \tag{31}$$

Joint Mean-Return Uncertainty

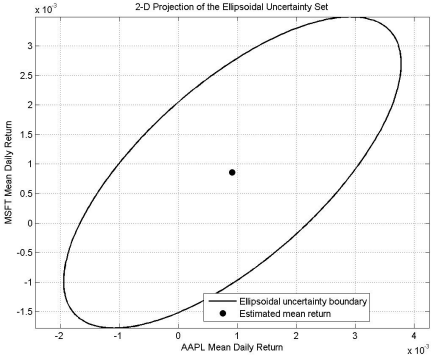


Figure 10: Two-dimensional projection of the ellipsoidal uncertainty set for AAPL and MSFT

Ellipsoidal Uncertainty: Three-Asset Projections

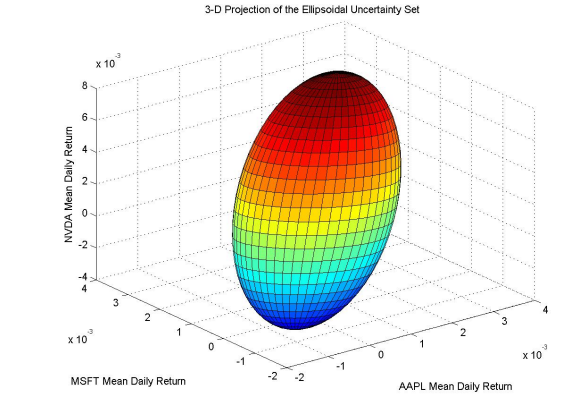
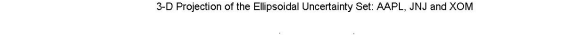


Figure 11: Ellipsoidal uncertainty set for AAPL, MSFT and NVDA.



Joint Uncertainty

- The 3D projections illustrate uncertainty in the **mean returns of three assets simultaneously**.
- Unlike the Box set, changes in one asset's estimated return can be considered together with changes in the other assets.

Key Interpretation

The ellipsoidal set captures the **joint behaviour of estimation errors**, providing a more flexible representation of uncertainty than independent

Ellipsoidal-Robust Markowitz: Optimal Portfolio

Table 10: Numerical Solution

Measure	Value
Worst-case Daily Return	-0.0000459090
Portfolio Variance	0.0000876681
Portfolio Std. Deviation	0.0093631252
Sum of Weights	1.0000000000

Table 11: Optimal Portfolio Weights

Asset	Weight	Asset	Weight
JNJ	16.6417%	WMT	16.1860%
KO	12.4586%	MCD	11.3499%
PG	10.3978%	MRK	10.0639%
COST	6.8221%	XOM	5.9478%
AMZN	5.0725%	ABBV	3.3674%
CAT	0.4693%	NEE	0.7428%
DIS	0.2588%	LLY	0.1637%
GOOGL	0.0580%	Other	0.0000%

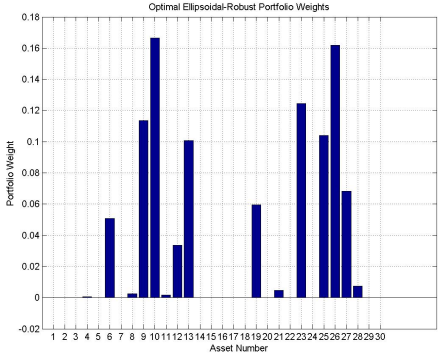


Figure 12: Optimal portfolio weights under ellipsoidal uncertainty.

Interpretation

The portfolio is concentrated mainly in **JNJ,**

Ellipsoidal-Robust Markowitz: Performance Measures

Table 12: Performance Measures

Measure	Value
Mean Daily Return	0.0003634260
Portfolio Variance	0.0000876681
Portfolio Std. Deviation	0.0093631252
Daily Risk-Free Rate	0.0001005000
Sharpe Ratio	0.028081
Sortino Ratio	0.054111
Omega Ratio	1.124463
95% VaR	0.0133163663
95% CVaR	0.0220700667
Maximum Drawdown	0.264201

Interpretation

The portfolio experienced a **26.42% maximum decline from a previous peak**, followed by recovery.

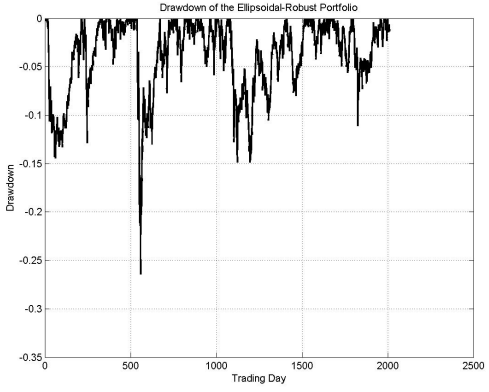


Figure 13: Drawdown of the optimal Ellipsoidal-Robust portfolio.

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THANK YOU !

Ondapandula!

Siyabonga !

Tangi unene!

Nagode!