Dynamic Bilevel Optimisation for Carbon Emission Reduction

Mr. Ali M.A. Haroon[1], Dr. Jean Medard T. Ngnotchouye[1], Prof. Surafel Luleseged Tilahun[2]

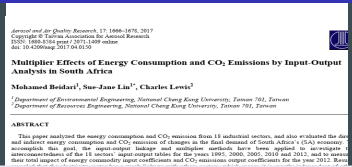
[1]Disciplines of Mathematics, School of Agriculture and Science, University of KwaZulu-Natal, Pietermaritzburg, South Africa
[2]Department of Mathematics, HPC and Big Data Analytics Center of Excellence, Addis Ababa Science and Technology University, Ethiopia

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Outline

1. Introduction & Literature Review
2. Aims and Objectives
3. Model Formulation
4. Theoretical Foundations
5. Adaptive Genetic Algorithm
6. Case Study
7. Conclusion

Introduction & Literature Review


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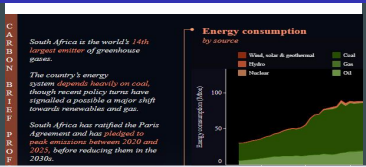
Multiplier Effects of Energy Consumption and CO_2 Emissions by Input-Output Analysis in South Africa

Mohamed Beidari[1], Sue-Jane Lin[1*], Charles Lewis[2]

[1] Department of Environmental Engineering, National Cheng Kung University, Tainan 701, Taiwan
[2] Department of Resources Engineering, National Cheng Kung University, Tainan 701, Taiwan

ABSTRACT

This paper analyzed the energy consumption and CO_2 emission from 18 industrial sectors, and also evaluated the dir... and indirect energy consumption and CO_2 emission of changes in the final demand of South Africa's (SA) economy. ... accomplish this goal, the input-output linkage and multiplier methods have been applied to ... investigate ... interconnectedness of the 18 sectors' input-output tables for the years 1995, 2000, 2005, 2010 and 2012, and to assess ... their total impact of energy commodity input coefficients and CO_2 emissions output coefficients for the year 2012. Rank...


CARBON BRIEF PROF

South Africa is the world's 14th largest emitter of greenhouse gases.

The country's energy system depends heavily on coal, though recent policy turns have signalled a possible a major shift towards renewables and gas.

South Africa has ratified the Paris Agreement and has pledged to peak emissions between 2020 and 2025, before reducing them in the 2030s.

Energy consumption
by source

Wind, solar & geothermal
Hydro
Nuclear
Coal
Gas
Oil

Energy consumption (EJ)
100
50
0


research article

climate and development

Changing development paths: From an energy-intensive to low-carbon economy in South Africa

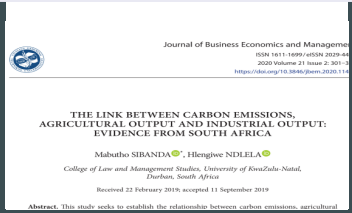
HARALD WINKLER* AND ANDREW MARQUAND

Energy Research Centre, University of Cape Town, Rondebosch 7701, South Africa

Climate change mitigation poses significant challenges for South Africa and its energy development, historically highly energy intensive. At the same time, the country faces a host of daunting development challenges, exacerbated by the legacy of apartheid. Examining both challenges, this paper considers how alternative conceptions of a development path can be achieved.

- Strategic play between carbon emissions, energy sector dynamics, and taxation policy is a critical environmental and economic challenge (Driscoll et al., 2020; Wu et al., 2022).

- In South Africa, a sector-wide analysis of 18 industrial sectors by Beidari et al. (2017) revealed that the electricity sector stands out as the most dominant source in terms of both energy and CO_2 intensities.


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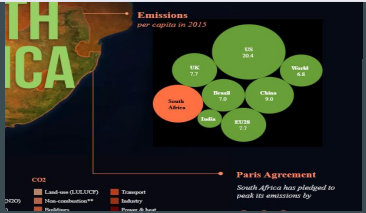
THE LINK BETWEEN CARBON EMISSIONS, AGRICULTURAL OUTPUT AND INDUSTRIAL OUTPUT: EVIDENCE FROM SOUTH AFRICA

Mabutho SIBANDA*, Hlengiwe NDLELA

College of Law and Management Studies, University of KwaZulu-Natal, Durban, South Africa

Received 22 February 2019; accepted 11 September 2019

Abstract. This study seeks to establish the relationship between carbon emissions, agricultural


TH ICA

Emissions
per capita in 2015

US 20.4
UK 7.7
World 4.8
Brazil 7.0
China 9.0
South Africa
India
EU28 7.7

Paris Agreement
South Africa has pledged to peak its emissions by

CO2
Land-use (LULUCF)
Non-combustion**
Buildings
Transport
Industry
Power & heat


SAJE South African Journal of Economics

South African Journal of Economics Vol. 81:5 September 2013

MEASURING THE CARBON INTENSITY OF THE SOUTH AFRICAN ECONOMY

CHANNING ARNDT[1], ROB DAVIES[2], KONSTANTIN MAKRELOV[3] AND JAMES THURLOW[4]

Abstract

We estimate the carbon intensity of industries, products and households in South Africa using data from a high resolution supply-use table. Direct and indirect carbon usage is measured using multiplier methods that capture inter-industry linkages and multi-product supply chains. Carbon intensity is found to be high for exports but low for major employing sectors. Middle-income households are the most carbon-intensive consumers. These results suggest that carbon pricing

Literature Review: Bilevel Applications

Our study builds upon a robust foundation of hierarchical modeling in environmental policy:

Electricity Pricing

Seminal frameworks established by Garg et al. (2020) successfully internalize external emission costs.

Policy Allocations

- Emission permit allocations (Feng et al., 2018)
- Energy-water nexus management (Zhang et al., 2016)
- High-polluting emission quotas (Huang et al., 2020)

Taxation & Strategy

- Optimizing static carbon taxes (Olsen et al., 2018)
- Clean production pathways balancing emission caps with profitability (Tsai et al., 2018)

Aims and Objectives

Main Aim

To design an effective and dynamic carbon policy framework that balances environmental and economic goals.

Specific Objectives

- Develop a dynamic bilevel model for carbon taxation.
- Design a specialised Adaptive Genetic Algorithm (GA) to solve it.
- Model the co-evolution between government policy and producer reactions.

Model Formulation

Assumptions .1

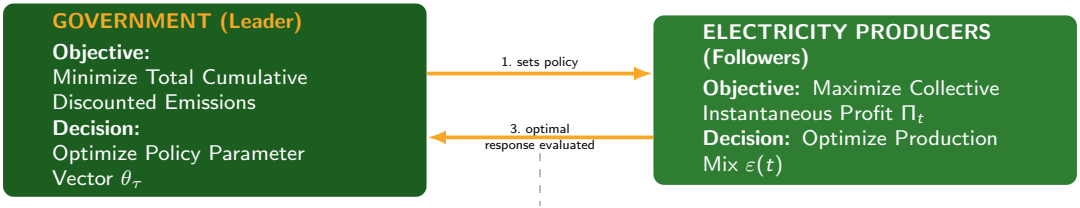
To build our model, we make a few simple, important assumptions.

- **Stackelberg Game** — leader (government) moves first; followers (producers) respond optimally.
- **Energy Sources** — classified by carbon intensity and cost (see table).
- **Dynamic Policies** — tax and subsidy instruments evolve over time.
- **Optimistic View** — the follower selects the leader-preferred response when indifferent.

Cat.	Examples	Emission	Cost
HCE	Coal	High	Low
MCE	Nat. Gas, Oil	Medium	Medium
LCE	Renewables, Nuclear	Low/Zero	High

Model Formulation

The Dynamic Bilevel Model (Stackelberg Game)



Model Formulation

Dynamic Policy Instruments: Tax Functions

HCE Tax $\tau_H(t)$

Exponential escalation — creates growing pressure for a phase-out of high-carbon sources.

$$\tau_H(t;\theta_\tau) = \gamma_1 e^{\alpha_1 t} + \beta_1 \varepsilon_H(t)^2$$

where $\gamma_1, \alpha_1, \beta_1 > 0$. The exponential term dominates over time, driving HCE production toward zero .

MCE Tax $\tau_M(t)$

Logistic function, scaled by production — manages the transition of intermediate sources.

$$\tau_M(t;\theta_\tau) = \tau_{\max}\left(\beta_2 - \frac{\gamma_2}{1+e^{-t/\alpha_2}}\right)\frac{\varepsilon_M(t)}{\varepsilon_{\max}}$$

where $\tau_{\max}, \varepsilon_{\max} > 0$ and $\beta_2, \gamma_2, \alpha_2 > 0$.

Model Formulation

Dynamic Policy Instruments: Subsidy & Revenue

LCE Subsidy $S(t)$

A decaying incentive, strongest when market penetration is low — supports technology maturation.

$$S(t;\theta_\tau) = S_0 e^{-\beta_3 t}\left(1 - \frac{\varepsilon_L(t)}{\varepsilon_{\max}}\right)$$

where system constant $S_0 > 0$ and parameter $\beta_3 > 0$.

Tax Revenue TR_t

The total tax revenue collected by the government at time t is the production-weighted sum of taxes.

$$TR_t(\tau(t), \varepsilon(t)) = \varepsilon_H(t)\tau_H(t) + \varepsilon_M(t)\tau_M(t)$$

Revenue funds the LCE subsidy under the revenue-neutrality constraint .

Model Formulation

The Dynamic Bilevel Optimisation Problem

Leader: $\min_{\theta_\tau \in \Theta} J(\theta_\tau) = \sum_{t=0}^{T} \delta^t \left(r_H \varepsilon_H^*(t) + r_M \varepsilon_M^*(t) + r_L \varepsilon_L^*(t) \right)$

s.t. $TR_t(\tau(t), \varepsilon^*(t)) \geq S(t) \varepsilon_L^*(t), \ \forall t \in T$ (revenue-neutrality)

Follower: $\varepsilon^*(t)$ solves

$\max_{\varepsilon(t)} \Pi_t(\tau(t), \varepsilon(t)) = P \sum_i \varepsilon_i(t) - \sum_i c_i \varepsilon_i(t) - TR_t(\tau(t), \varepsilon^*(t)) + \varepsilon_L(t) S(t)$

s.t. $\sum_i \varepsilon_i(t) \leq \varepsilon_{\max}, \quad \varepsilon_i(t) \geq 0, \quad i \in \{H, M, L\}$

where $\delta \in (0, 1]$ is the discount factor and $r_H > r_M > r_L \geq 0$ are technology-specific emission rates.

The aggregate constraint $\sum_i \varepsilon_i(t) \leq \varepsilon_{\max}$ shown above is too loose on its own, it lets, e.g., MCE production exceed its entire real fleet. Every numerical result in this deck instead uses per-technology capacity limits $0 \leq \varepsilon_i(t) \leq \mathrm{cap}_i$ for $i \in \{H, M, L\}$.

Theoretical Foundations

Core Bilevel Concepts

For the lower-level problem at a given time t and for a fixed policy $\tau(t)$ (Schmidt et al., 2023):

Def. 1: Feasible Set

The compact, convex set of feasible production mixes:

$$F = \left\{ \varepsilon \in \mathbb{R}^3_+ \;\middle|\; \sum_i \varepsilon_i \leq \varepsilon_{\max} \right\}$$

Def. 2: Rational Reaction Set

The set of optimal lower-level responses:

$$R(\theta_\tau, t) = \underset{\varepsilon \in F}{\operatorname{argmax}} \{ \Pi_t(\tau(t; \theta_\tau), \varepsilon) \}$$

Def. 3: Inducible Region

Outcomes achievable by the leader, respecting the follower's rational response:

$$IR = \left\{ (\theta_\tau, \varepsilon) \;\middle|\; \exists \{\varepsilon(t)\}_{t=0}^{T}, \; \varepsilon(t) \in R(\theta_\tau, t) \; \forall t \right\}$$

Assumptions .2: Regularity Conditions

For the problem to be well-posed, we assume:

- **Compactness.** The leader's parameter space Θ is a non-empty, compact subset of a finite-dimensional Euclidean space.
- **Continuity.** All objective functions (J, Π_t) and constraint functions are continuous on their respective domains.
- **Unique Follower Response.** For any feasible leader policy $\theta_\tau \in \Theta$, the follower's rational reaction set $R(\theta_\tau, t)$ is a singleton for all t.

Theoretical Foundations

Lemma: Strict Concavity of the Follower's Problem (Haroon, Ngnotchouye and Tilahun 2026)

Let the leader's policy parameters be constrained such that $\beta_1 > 0$, and the condition $\beta_2 > \gamma_2$ holds. Then for any $t \in \mathbb{T}$, the follower's profit function Π_t is strictly concave with respect to ϵ over the interior of the feasible set $\mathcal{F}$.

Proof

For Π_t to be strictly concave, its Hessian matrix must be negative definite. Since the Hessian is diagonal, this is equivalent to all its diagonal entries being strictly negative.

- The first entry, $-6\beta_1\epsilon_H$, is strictly negative for any $\epsilon_H > 0$, as $\beta_1 > 0$. At the boundary where $\epsilon_H = 0$, this term is zero, indicating potential semi-definiteness. However, since the objective contains a term $-\beta_1\epsilon_H^3$, the function $f_H(\epsilon_H)$ is strictly concave for $\epsilon_H \geq 0$, which is sufficient for our purpose.

... Continue Proof

- The second entry is strictly negative if $\beta_2 > \frac{\gamma_2}{1+e^{-t/\alpha_2}}$. As $t \geq 0$, we have $1 < 1 + e^{-t/\alpha_2} \leq 2$, which implies $\frac{\gamma_2}{2} \leq \frac{\gamma_2}{1+e^{-t/\alpha_2}} < \gamma_2$. Therefore, the sufficient condition $\beta_2 > \gamma_2$ ensures the term is strictly negative for all t. - The third entry, $-2S_0 e^{-\beta_3 t}/\epsilon_{\text{max}}$, is strictly negative for all t because all its constituent constants $(S_0, \beta_3, \epsilon_{\text{max}})$ are positive. Since all conditions are met, $\nabla^2_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}} \Pi_t$ is negative definite over the interior of $\mathcal{F}$. Combined with the fact that $\mathcal{F}$ is a convex set, the existence of a unique global maximizer $\boldsymbol{\epsilon}^*(t)$ is guaranteed by standard results in convex optimisation.

Theoretical Foundations

Theorem 1: Follower's Optimal Response Function (Haroon, Ngnotchouye and Tilahun 2026)

For a given policy $\tau(t;\theta_\tau)$, the follower's unique optimal production mix $\varepsilon^*(t)$ solves $\partial\Pi_t/\partial\varepsilon_i = 0$. The unconstrained optimum for each source:

$$\varepsilon_H^{uc}(t) = \max\left\{0,\ \sqrt{\frac{P - c_H - \gamma_1 e^{\alpha_1 t}}{3\beta_1}}\right\}$$

$$\varepsilon_M^{uc}(t) = \max\left\{0,\ \frac{\varepsilon_{\max}(P - c_M)}{2\tau_{\max}\left[\beta_2 - \gamma_2/(1 + e^{-t/\alpha_2})\right]}\right\}$$

$$\varepsilon_L^{uc}(t) = \max\left\{0,\ \frac{\varepsilon_{\max}}{2}\left(1 + \frac{P - c_L}{S_0 e^{-\beta_3 t}}\right)\right\}$$

The constrained optimal response $\varepsilon^*(t)$ is the Euclidean projection of $\varepsilon^{uc}(t)$ onto $F = \{\varepsilon \in \mathbb{R}^3_+ \mid \sum_i \varepsilon_i \leq \varepsilon_{\max}\}$.

Theoretical Foundations

Theorem 1: Proof

- For Π_t to be strictly concave, its Hessian must be negative definite. Since the Hessian is diagonal, this reduces to each diagonal entry being strictly negative.

- This ensures the Karush–Kuhn–Tucker (KKT) conditions are both necessary and sufficient for optimality.

- Due to the separable structure of $\Pi_t(\varepsilon)$, the unconstrained first-order conditions yield the closed-form expressions on the previous slide, with $\max(0, \cdot)$ enforcing non-negativity.

- If $\sum_i \varepsilon_i^{uc} > \varepsilon_{\max}$, the optimum lies on the boundary of the capacity simplex. Since the objective is separable quadratic/cubic and the constraint linear, this is the projection of ε^{uc} onto F, solvable efficiently via a water-filling algorithm.

Theoretical Foundations

Theorem 2: Energy Transition Dynamics (Haroon, Ngnotchouye and Tilahun 2026)

MCE production is strictly increasing, in the unconstrained regime.

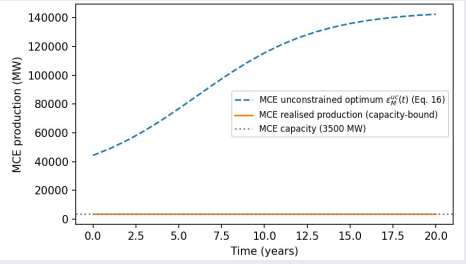
- **HCE decline** : $\dfrac{d\varepsilon_H^*}{dt} = -\dfrac{\alpha_1\gamma_1 e^{\alpha_1 t}}{6\beta_1\varepsilon_H^*(t)} < 0$

- **MCE growth** : $\dfrac{d\varepsilon_M^*}{dt} = \dfrac{\gamma_2\varepsilon_{\max}(P - c_M)e^{-t/\alpha_2}}{2\alpha_2\tau_{\max}\left[\beta_2(1 + e^{-t/\alpha_2}) - \gamma_2\right]^2} > 0$

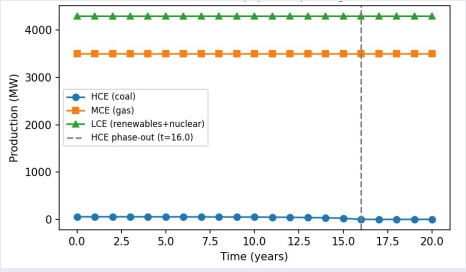
-**LCE growth** : $\dfrac{d\varepsilon_L^*}{dt} = \dfrac{\beta_3\varepsilon_{\max}(P - c_L)e^{\beta_3 t}}{2S_0} > 0$

Theoretical Foundations

Theorem 2: Evidence from the SA Calibration



The unconstrained MCE response is strictly. bout is capacity-bound in this calibration



ODP production mix, MCE and LCE grow after HCE phase-out

- *The unconstrained MCE response $\varepsilon_M^{uc}(t)$ is strictly increasing, but the realised, capacity-bound production stays flat at 3,500 MW.*
- *The ODP production mix shows MCE holding steady (at its cap) and gently rising after the HCE phase-out at $t \approx 16$ years.*

Theoretical Foundations

Theorem 3: Finite-Time Phase-Out of HCE Production (Haroon, Ngnotchouye and Tilahun 2026)

Assume the leader selects policy parameters such that $\alpha_1 > 0$ and the initial condition $(P - c_H) > \gamma_1$ holds. Then there exists a finite critical time, t_{crit}, after which HCE production ceases entirely: $\varepsilon_H^*(t) = 0$ for all $t \geq t_{crit}$.

$$t_{crit} = \frac{1}{\alpha_1} \ln\left(\frac{P - c_H}{\gamma_1}\right)$$

This result provides the leader with a direct, deterministic control lever over the phase-out timeline via α_1 and γ_1.

Theoretical Foundations

Theorem 3: Proof

- From the HCE response, $\varepsilon_H^{uc}(t) = 0$ when the term inside the square root becomes non-positive.
- The critical time t_{crit} is where this term equals zero: $P - c_H - \gamma_1 e^{\alpha_1 t_{crit}} = 0$.
- Solving: $e^{\alpha_1 t_{crit}} = (P - c_H)/\gamma_1 \implies t_{crit} = \frac{1}{\alpha_1} \ln[(P - c_H)/\gamma_1]$.
- The initial condition $(P - c_H) > \gamma_1$ ensures the log argument exceeds one, so $t_{crit} > 0$.
- For all $t > t_{crit}$, the exponential term dominates, making the numerator negative, so $\varepsilon_H^*(t) = 0$.

The leader controls the phase-out timeline directly via α_1 and γ_1.

Solving the Model: Adaptive Genetic Algorithm

Our algorithm's parameters evolve, just like the policies we are looking for.

Dynamic Crossover Rate (p_c)

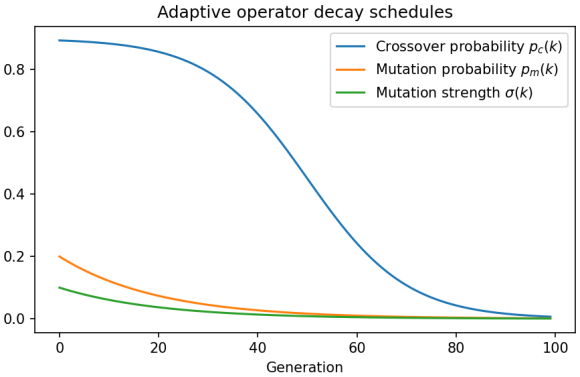
$$p_c(k) = \frac{p_{c,0}}{1 + e^{\lambda(k - k_{mid})}}$$

- Start: high probability → explore the search space with big changes.
- End: low probability → fine-tune the best solutions found.

Dynamic Mutation Rate (p_m)

$$p_m(k) = p_{m,0}\, e^{-\delta k}$$

- Start: high chance of mutation → avoids getting stuck in bad solutions.



Actual decay schedules used in the South Africa run (crossover probability, mutation probability, mutation strength).

Case Study

Case Study Context: South Africa

The model is calibrated using parameters reflecting South Africa's coal-dependent power sector (representative of Eskom's generation landscape).

Metric / Parameter	HCE (Coal)	MCE (Gas)	LCE (Renew./Nuclear)
Technology Type	Coal-Fired	Gas (OCGT)	Solar, Wind, Nuclear
Production Cost (c_i)	\$23.10/MWh	\$137.50/MWh	\$165.00/MWh
Capacity Limit	35,000 MW	3,500 MW	4,300 MW
Emission Rate (r_i)	0.95 tCO_2/MWh	0.50 tCO_2/MWh	0.05 tCO_2/MWh

System constants: $\varepsilon_{\max} = 42{,}800\,MW \cdot P = \$190/MWh \cdot \tau_{\max} = \$40/tCO_2 \cdot S_0 = \$80/MWh \cdot \delta = 0.95$

Case Study

Case Study Context: South Africa | Policy Parameter Search Space for the GA

Parameter	Search Bounds
α_1 (Tax escalation rate)	$[0.05, 0.15]$
β_1 (Production penalty coeff.)	$[0.01, 0.05]$
γ_1 (Base tax rate, USD/tCO$_2$)	$[10, 30]$
α_2 (Transition timescale, yrs)	$[2, 7]$
β_2 (Initial tax rate scaler)	$[0.3, 0.6]$
γ_2 (Tax decay magnitude)	$[0.2, 0.4]$
β_3 (Subsidy decay rate)	$[0.10, 0.30]$

Comparative Baselines

We compare the Optimal Dynamic Policy (ODP) against:

- **Optimal Static Policy (OSP)** — escalation parameters fixed at zero
- **No-Subsidy Policy (NSP)** — as OSP, but $S_0 = 0$

System constants: $P = \$190/MWh \cdot \varepsilon_{\max} = 42{,}800\,MW \cdot S_0 = \$80/MWh \cdot \delta = 0.95$

Case Study

Case Study Context: South Africa | Policy Scenario Comparison: 7-Runs Recompute

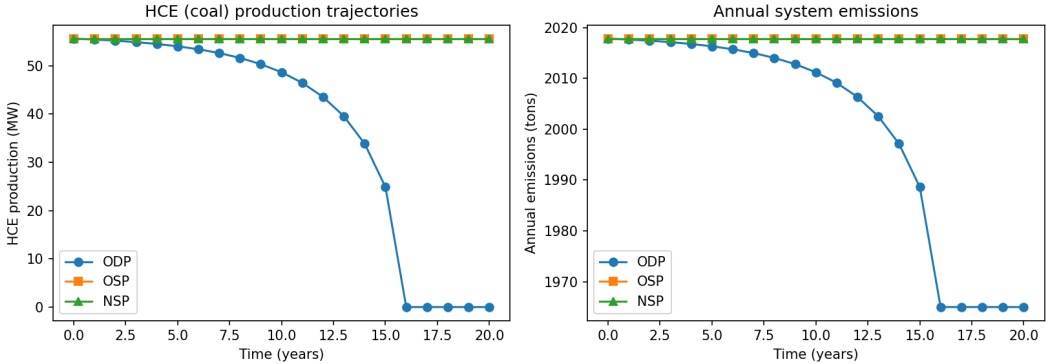
ROBUST: structural result

ODP is the only policy that reliably achieves a full HCE phase-out — 4 of 7-Runs (mean year 15.25). OSP and NSP achieve phase-out in 0 of 7-Runs.

Scenario	Mean Emis. (tons)	% Spread vs. ODP	Phase-out achieved
ODP	26,459	—	**4/7 runs** (mean yr 15.25)
OSP	26,566	0.46% (0.09–0.91%)	0/7 runs
NSP	26,566	0.46% (identical to OSP)	0/7 runs

Case Study

Case Study Context: South Africa | Policy Scenario Comparison: Representative Trajectory



Single representative run. HCE production under the ODP declines to zero around $t \approx 16$ years while OSP/NSP stay near their static ceiling; annual emissions fall correspondingly. The 7-Run statistics on the previous slide are the basis for all headline claims — this chart illustrates the qualitative pattern.

Case Study Context: Poland | A Second Coal-Dependent Economy

- **Why Poland:** the 5th-highest 2025 coal share of generation among covered economies (51.1%), after South Africa (78.8%), India (71.2%), Indonesia (61.0%), and China (54.4%).
- **Structurally comparable** to South Africa: as of 2019–2020, Poland lacked domestic gas infrastructure for coal-to-gas switching — the same no-fuel-switch dispatch context.
- Added as an additional scenario-matrix row, not a replacement for South Africa.

Mixed evidentiary standard: every parameter below is tagged Sourced, Derived, or Proxy — none are fabricated. Capacity, market price, and coal/solar costs are directly sourced; gas cost, per-technology emission rates, and subsidy level rest on shown derivations or proxies.

Parameter	Value	Status
Capacity (HCE/MCE/LCE)	26.5 / 5.9 / 37.1 GW	Sourced
Market price P	115.15 USD/MWh	Sourced
c_H (coal, carbon-excl.)	44.69 USD/MWh	Sourced
c_M (gas, carbon-excl.)	91.98 USD/MWh	Derived
c_L (solar only)	47.42 USD/MWh	Sourced
S_0 (subsidy level)	103.19 USD/MWh	Proxy

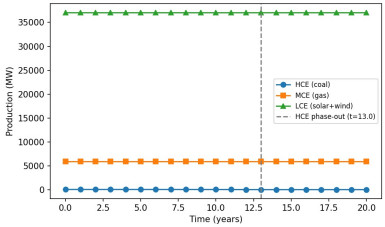
Case Study

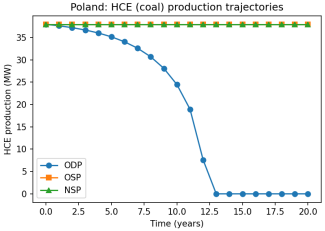
Case Study Context: Poland | Results & a Real Cost-Ordering Inversion

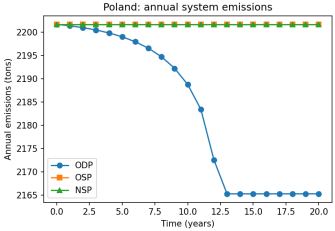
0.63%
ODP margin vs. static

13.0 yrs
HCE phase-out under ODP, neither static gets there

$c_L < c_M$
Solar now cheaper than gas, even before carbon pricing







Case Study

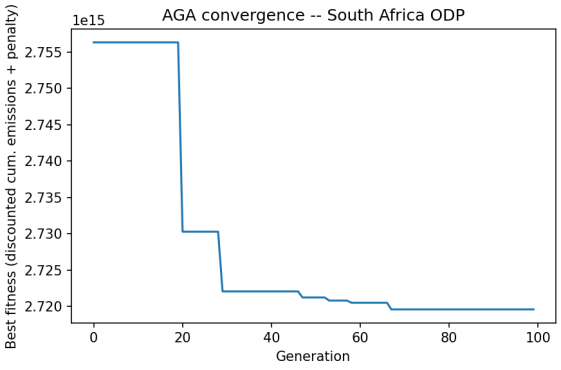
Case Study Context: Poland | A Real Finding, Not Smoothed Over

$c_H \approx 44.7$, $c_M \approx 92.0$, $c_L \approx 47.4$ USD/MWh — i.e. $c_L < c_M$, violating the model's assumed cost ordering $c_H < c_M < c_L$. This reflects a documented 2025 Polish/European phenomenon (unsubsidised solar undercutting gas SRMC even before carbon pricing), not a data error. Nothing in the follower's first-order conditions requires the ordering to hold, so the model runs unmodified — c_L is not adjusted upward.

- Reported as a single-run, mixed-evidentiary-standard illustration — not a like-for-like replication of South Africa's 7-Runs comparison.
- AGA search bounds inherited unchanged from South Africa (not independently re-tuned); revenue-neutrality penalty at convergence was larger (16.2M) than for South Africa.
- A multi-Run replication with re-tuned bounds, sourced wind LCOE, and technology-level emission rates is left for future work.

Algorithmic Performance & Convergence

- The proposed Adaptive Genetic Algorithm (AGA) shows rapid initialization, locating high-quality policy parameter vectors within the first $\sim$20 generations.
- The convergence curve stabilizes smoothly, demonstrating the operational validity of the elitism-preserving operator schedules.



Conclusion

ROBUST: what the model reliably delivers

Bilevel, dynamic policy design reliably achieves a finite-time HCE phase-out. Static, one-size-fits-all policies do not, in any tested run.

MODEST: hedged, evidence-proportionate framing

On the emissions-level metric itself, the ODP is competitive with, not dramatically better than, static policy (a small, run-robust margin, $\sim$0.46%).

- Bilevel optimisation remains a valuable framework for policy design, the leader's parameters $(\alpha_1, \gamma_1, \beta_3, \ldots)$ function as explicit, interpretable control levers for transition speed.

Dynamic Bilevel Optimisation for Carbon Emission Reduction: Integrating Nonlinear Policy Functions and Adaptive Genetic Algorithms

Ali M.A. Haroon, Jean Medard T. Ngnotchouye, Surafel Luleseged Tilahun

***Abstract*—This paper presents a dynamic bilevel optimisation framework for designing and evaluating carbon emission reduction policies. It models the strategic interaction between a policymaker (the leader), who seeks to minimize emissions, and profit-maximizing energy producers (the followers). We introduce nonlinear tax and subsidy functions that evolve over time, enabling a more realistic simulation of escalating policy interventions. Energy production technologies are classified into three categories: high-carbon, ed on their carbon r theoretical analysis nd, most importantly, ents can guarantee a sources. To solve the etic algorithm with on parameters. We calibrated with data results show that the outperforms static ation by engineering a ng system constraints. and robust decision esign and calibrate**

targets. Carbon taxation is widely recognized as an effective market-based mechanism for internalizing the externalities of electricity generation [5]–[7], given the well-established positive correlation between energy production and carbon dioxide (CO_2) emissions [8]–[10].

This study models this complex interaction as a dynamic bilevel optimisation problem (DBOP). In this Stackelberg game-theoretic framework, the government acts as the upper-level (leader), setting carbon tax rates over a multi-period horizon to minimize cumulative, system-wide emissions. The lower-level (followers) consists of competing electricity producers who, in response to the leader's policy, seek to maximize their individual profits by dynamically adjusting their generation portfolio across high-carbon emission (HCE), medium-carbon emission (MCE), and low-carbon emission (LCE) sources. A key feature of our model is a revenue-neutrality constraint, which mandates that tax revenues from HCE and MCE producers are recycled as subsidies to LCE producers, thereby creating a direct financial incentive for energy transition.

Our study builds upon a substantial body of literature applying bilevel optimisation to energy and environmental policy. Seminal works, such as Garg et al. [11], established frameworks for electricity pricing that internalize emission costs. Subsequent models have addressed diverse policy challenges, including emissions permit allocation [12], integrated management of the energy-water nexus [13], and the imposition of emission quotas for high-polluting sectors [14]. Other related studies have focused on determining optimal static carbon taxes [15] and integrating modern production technologies to balance emission caps with profitability [16]. However, a limitation of many existing models is their reliance on static, single-period analyses or linear policy functions. This work advances the literature by introducing dynamic, nonlinear tax and subsidy functions that evolve based on both time and aggregate production levels, transforming the equilibrium problem into a more realistic, time-dependent optimal control challenge inspired by advanced subsidy designs [17].

This paper makes three primary contributions to the field of environmental policy optimisation.

1) We formalize dynamic, time-dependent nonlinear tax and subsidy functions that capture the complexities of real-world escalating policy interventions. We go beyond descriptive modeling by theoretically characterizing the system's monotonic dynamics and providing a formal proof that this policy structure guarantees a finite-time phase-out of HCE sources,

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Displacement-Aware Multi-Fuel Dynamics in Dynamic Bilevel Carbon Policy Optimisation★

Ali M. A. Haroon[a,c,*], Jean Medard T. Ngnotchouye[a] and Surafel Luleseged Tilahun[b]

[a]*Disciplines of Mathematics, School of Agriculture and Science, University of KwaZulu-Natal, Scottsville, Private Bag X01, Pietermaritzburg, South Africa.*

[b]*Department of Mathematics, HPC and Big Data Analytics Center of Excellence, Addis Ababa Science and Technology University, 16417, Addis Ababa, Ethiopia.*

[c]*Department of Mathematics, University of Kordofan, El-Obeid, 51111, North Kordofan, Sudan*

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ABSTRACT

This study extends the dynamic carbon emission model (DBOP), in which a government leader sets nonlinear, time-escalating tax and subsidy instruments, and profit-maximising producers respond by choosing a production mix across high-, medium-, and low-carbon-emission (HCE, MCE, LCE) sources. Their model proves a finite-horizon phase-out of HCE production and characterises the resulting monotonic transition dynamics, but does so under an implicit assumption that the three production streams respond independently to policy — nothing in the follower's problem lets a reduction in one fuel's output reappear as an increase in another's. This paper relaxes that assumption. We introduce cross-fuel displacement elasticities and a waterbed constraint that couple HCE, MCE and LCE production under a fixed emissions cap, decompose the follower's response into substitution and output effects, disaggregate production by sector, and formalise a subsidy-leakage mechanism linking domestic incentives to market integration. We further extend the original Adaptive Genetic Algorithm into a multi-objective, displacement-aware variant that traces the Pareto frontier for emissions

1. Introduction

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Thank You

Questions?